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MTH360 – Project 2

For our project, we decided to solve an eight week, eight team season with each team getting one bye. Each $x_{i,j,k}$ variable corresponds to a game between teams i and j during week k . We developed a sample matrix B that represents which teams have a bye on which weeks. The sets of constraints, `ByeWeeks1` and `ByeWeeks2`, loop through this array and do not allow a game to occur with any particular team that has a bye week. We need both sets because of the symmetrical nature of our x variables. We want to constrain games where team 1 is playing team 3 and also where team 3 is playing team 1. This might seem redundant, but this can cause issues because of how we defined x .

Our “More” constraint set also handles symmetry by making certain, for example, that $x_{1,2,1} = x_{2,1,1}$. The reason this is important is because we do not want duplicate games in the schedule. The LP will treat each variable as its own entity so if we do not make this distinction, then more games will be scheduled than necessary, which could skew the solution or even make infeasible from a practical standpoint.

Our next two sets on constraints, `HTConstraints` and `VTConstraints`, ensure that any particular team can only play a maximum of one game per week. For example,

$$x[1, 1, 1] + x[1, 2, 1] + x[1, 3, 1] + x[1, 4, 1] + x[1, 5, 1] + x[1, 6, 1] + x[1, 7, 1] + x[1, 8, 1] \leq 1$$

tells us that team 1 can play any other team during week 1, but the sum of those games must equal 1. This makes sure that either 0 or 1 game is played. The other set is the symmetrical version

$x[1,1,1] + x[2,1,1] + x[3,1,1] + x[4,1,1] + x[5,1,1] + x[6,1,1] + x[7,1,1] + x[8,1,1] \leq 1$, which says that team 1 can only be challenged by at most 1 other team in a given week. This is a subtle distinction, but necessary nonetheless. We need to be sure that any given team is only playing and being played one time per week.

`TConstraints2` is a simple set of constraints to ensure that a team cannot play itself on any given week. This constraint could change depending on the nature of the sport, but for this project it was not allowed.

Our objective function is simply the sum of all x variables in the problem. Most of these will equal zero, but the sum divided by two will represent the total number of games played in the season because there is a variable for representing a game for each team. See the attached Maple worksheet for the decision variable values. The loop at the end of worksheet creates matrices for each week and shows which teams play for that week.

We noticed that when solving this problem using the regular LPSolve command, it will complete in a matter of seconds. It will give exact values for the highest possible z-value, which is great, but unfortunately, we have not found a good way to play 60% of a game. So when we switch it to an integer linear programming problem, we could just round the LP solution and hope it is still both a maximization and inside the feasible region, but this is usually not the case. A lot of the times the ILP solution will be much different than the LP. This is why the algorithm takes so much longer to complete when dealing with large ILPs. We learned in class about the branch and bound method that will iterate through almost every possible integer solution until it finds the optimal one or at least the best sub-optimal possible. When it completes this it gives a very nice solution set of whole numbers. This is better than LP sometimes because it can spit out really close estimates instead of whole numbers. In conclusion, the main difference we saw was the time difference to execute the algorithm and the solutions were quite different. The LP solution does not make sense in the context of the problem while the ILP solution definitely does.

Matt Conflitti Narrative

I now understand why mathematicians can get paid very well to solve problems like these efficiently. This problem is very small compared to some of the route and trip scheduling work they did at my past internship. Before this class, it all seemed like magic, but now I see how important operations research is and can be for any industry. Specifically in this project, I learned that ILPs can be much more complex than LPs. I am sure with dedicated hardware to solve ILPs it wouldn't take so long to compute, but as a computer science major as well it was cool to see complex algorithms in practice outside of a programming class. I guess I also learned that if a problem can be represented as an LP and not an ILP then do that. It is not worth the extra overhead unless absolutely necessary. Overall solid project with a very good real life application.

```
> with(LinearAlgebra): with(Optimization):
> n:=8; m:=8; #set number of teams and weeks
      n := 8
      m := 8
```

(1)

```
> x:=array(1..n,1..n,1..m);
      x := array(1..8,1..8,1..8, [ ])
>
```

(2)

```
> bMatrix := <<1,1,0,0,0,0,0,0>|<0,0,0,0,0,0,0,0>|<0,0,0,0,1,1,0,0>|
0>|<0,0,0,0,0,0,1,1>|<0,0,0,0,0,0,0,0>|<0,0,0,0,0,0,0,0>|<0,0,0,0,0,0,0,0>|<0,0,1,1,0,0,0,0>>;
```

$$bMatrix := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(3)

```
> ByeWeeks1 := seq(seq(seq(x[i,j,k]<=abs(bMatrix[i,k]-1), i=1..n), j=1..n), k=1..m);
```

(4)

$$\begin{aligned} \text{ByeWeeks1} := & x_{1,1,1} \leq 0, x_{2,1,1} \leq 0, x_{3,1,1} \leq 1, x_{4,1,1} \leq 1, x_{5,1,1} \leq 1, x_{6,1,1} \leq 1, x_{7,1,1} \\ & \leq 1, x_{8,1,1} \leq 1, x_{1,2,1} \leq 0, x_{2,2,1} \leq 0, x_{3,2,1} \leq 1, x_{4,2,1} \leq 1, x_{5,2,1} \leq 1, x_{6,2,1} \\ & \leq 1, x_{7,2,1} \leq 1, x_{8,2,1} \leq 1, x_{1,3,1} \leq 0, x_{2,3,1} \leq 0, x_{3,3,1} \leq 1, x_{4,3,1} \leq 1, x_{5,3,1} \\ & \leq 1, x_{6,3,1} \leq 1, x_{7,3,1} \leq 1, x_{8,3,1} \leq 1, x_{1,4,1} \leq 0, x_{2,4,1} \leq 0, x_{3,4,1} \leq 1, x_{4,4,1} \\ & \leq 1, x_{5,4,1} \leq 1, x_{6,4,1} \leq 1, x_{7,4,1} \leq 1, x_{8,4,1} \leq 1, x_{1,5,1} \leq 0, x_{2,5,1} \leq 0, x_{3,5,1} \\ & \leq 1, x_{4,5,1} \leq 1, x_{5,5,1} \leq 1, x_{6,5,1} \leq 1, x_{7,5,1} \leq 1, x_{8,5,1} \leq 1, x_{1,6,1} \leq 0, x_{2,6,1} \\ & \leq 0, x_{3,6,1} \leq 1, x_{4,6,1} \leq 1, x_{5,6,1} \leq 1, x_{6,6,1} \leq 1, x_{7,6,1} \leq 1, x_{8,6,1} \leq 1, x_{1,7,1} \\ & \leq 0, x_{2,7,1} \leq 0, x_{3,7,1} \leq 1, x_{4,7,1} \leq 1, x_{5,7,1} \leq 1, x_{6,7,1} \leq 1, x_{7,7,1} \leq 1, x_{8,7,1} \\ & \leq 1, x_{1,8,1} \leq 0, x_{2,8,1} \leq 0, x_{3,8,1} \leq 1, x_{4,8,1} \leq 1, x_{5,8,1} \leq 1, x_{6,8,1} \leq 1, x_{7,8,1} \\ & \leq 1, x_{8,8,1} \leq 1, x_{1,1,2} \leq 1, x_{2,1,2} \leq 1, x_{3,1,2} \leq 1, x_{4,1,2} \leq 1, x_{5,1,2} \leq 1, x_{6,1,2} \\ & \leq 1, x_{7,1,2} \leq 1, x_{8,1,2} \leq 1, x_{1,2,2} \leq 1, x_{2,2,2} \leq 1, x_{3,2,2} \leq 1, x_{4,2,2} \leq 1, x_{5,2,2} \\ & \leq 1, x_{6,2,2} \leq 1, x_{7,2,2} \leq 1, x_{8,2,2} \leq 1, x_{1,3,2} \leq 1, x_{2,3,2} \leq 1, x_{3,3,2} \leq 1, x_{4,3,2} \\ & \leq 1, x_{5,3,2} \leq 1, x_{6,3,2} \leq 1, x_{7,3,2} \leq 1, x_{8,3,2} \leq 1, x_{1,4,2} \leq 1, x_{2,4,2} \leq 1, x_{3,4,2} \\ & \leq 1, x_{4,4,2} \leq 1, x_{5,4,2} \leq 1, x_{6,4,2} \leq 1, x_{7,4,2} \leq 1, x_{8,4,2} \leq 1, x_{1,5,2} \leq 1, x_{2,5,2} \\ & \leq 1, x_{3,5,2} \leq 1, x_{4,5,2} \leq 1, x_{5,5,2} \leq 1, x_{6,5,2} \leq 1, x_{7,5,2} \leq 1, x_{8,5,2} \leq 1, x_{1,6,2} \end{aligned}$$

$$\begin{aligned}
&\leq 1, x_{2,6,2} \leq 1, x_{3,6,2} \leq 1, x_{4,6,2} \leq 1, x_{5,6,2} \leq 1, x_{6,6,2} \leq 1, x_{7,6,2} \leq 1, x_{8,6,2} \\
&\leq 1, x_{1,7,2} \leq 1, x_{2,7,2} \leq 1, x_{3,7,2} \leq 1, x_{4,7,2} \leq 1, x_{5,7,2} \leq 1, x_{6,7,2} \leq 1, x_{7,7,2} \\
&\leq 1, x_{8,7,2} \leq 1, x_{1,8,2} \leq 1, x_{2,8,2} \leq 1, x_{3,8,2} \leq 1, x_{4,8,2} \leq 1, x_{5,8,2} \leq 1, x_{6,8,2} \\
&\leq 1, x_{7,8,2} \leq 1, x_{8,8,2} \leq 1, x_{1,1,3} \leq 1, x_{2,1,3} \leq 1, x_{3,1,3} \leq 1, x_{4,1,3} \leq 1, x_{5,1,3} \\
&\leq 0, x_{6,1,3} \leq 0, x_{7,1,3} \leq 1, x_{8,1,3} \leq 1, x_{1,2,3} \leq 1, x_{2,2,3} \leq 1, x_{3,2,3} \leq 1, x_{4,2,3} \\
&\leq 1, x_{5,2,3} \leq 0, x_{6,2,3} \leq 0, x_{7,2,3} \leq 1, x_{8,2,3} \leq 1, x_{1,3,3} \leq 1, x_{2,3,3} \leq 1, x_{3,3,3} \\
&\leq 1, x_{4,3,3} \leq 1, x_{5,3,3} \leq 0, x_{6,3,3} \leq 0, x_{7,3,3} \leq 1, x_{8,3,3} \leq 1, x_{1,4,3} \leq 1, x_{2,4,3} \\
&\leq 1, x_{3,4,3} \leq 1, x_{4,4,3} \leq 1, x_{5,4,3} \leq 0, x_{6,4,3} \leq 0, x_{7,4,3} \leq 1, x_{8,4,3} \leq 1, x_{1,5,3} \\
&\leq 1, x_{2,5,3} \leq 1, x_{3,5,3} \leq 1, x_{4,5,3} \leq 1, x_{5,5,3} \leq 0, x_{6,5,3} \leq 0, x_{7,5,3} \leq 1, x_{8,5,3} \\
&\leq 1, x_{1,6,3} \leq 1, x_{2,6,3} \leq 1, x_{3,6,3} \leq 1, x_{4,6,3} \leq 1, x_{5,6,3} \leq 0, x_{6,6,3} \leq 0, x_{7,6,3} \\
&\leq 1, x_{8,6,3} \leq 1, x_{1,7,3} \leq 1, x_{2,7,3} \leq 1, x_{3,7,3} \leq 1, x_{4,7,3} \leq 1, x_{5,7,3} \leq 0, x_{6,7,3} \\
&\leq 0, x_{7,7,3} \leq 1, x_{8,7,3} \leq 1, x_{1,8,3} \leq 1, x_{2,8,3} \leq 1, x_{3,8,3} \leq 1, x_{4,8,3} \leq 1, x_{5,8,3} \\
&\leq 0, x_{6,8,3} \leq 0, x_{7,8,3} \leq 1, x_{8,8,3} \leq 1, x_{1,1,4} \leq 1, x_{2,1,4} \leq 1, x_{3,1,4} \leq 1, x_{4,1,4} \\
&\leq 1, x_{5,1,4} \leq 1, x_{6,1,4} \leq 1, x_{7,1,4} \leq 0, x_{8,1,4} \leq 0, x_{1,2,4} \leq 1, x_{2,2,4} \leq 1, x_{3,2,4} \\
&\leq 1, x_{4,2,4} \leq 1, x_{5,2,4} \leq 1, x_{6,2,4} \leq 1, x_{7,2,4} \leq 0, x_{8,2,4} \leq 0, x_{1,3,4} \leq 1, x_{2,3,4} \\
&\leq 1, x_{3,3,4} \leq 1, x_{4,3,4} \leq 1, x_{5,3,4} \leq 1, x_{6,3,4} \leq 1, x_{7,3,4} \leq 0, x_{8,3,4} \leq 0, x_{1,4,4} \\
&\leq 1, x_{2,4,4} \leq 1, x_{3,4,4} \leq 1, x_{4,4,4} \leq 1, x_{5,4,4} \leq 1, x_{6,4,4} \leq 1, x_{7,4,4} \leq 0, x_{8,4,4} \\
&\leq 0, x_{1,5,4} \leq 1, x_{2,5,4} \leq 1, x_{3,5,4} \leq 1, x_{4,5,4} \leq 1, x_{5,5,4} \leq 1, x_{6,5,4} \leq 1, x_{7,5,4} \\
&\leq 0, x_{8,5,4} \leq 0, x_{1,6,4} \leq 1, x_{2,6,4} \leq 1, x_{3,6,4} \leq 1, x_{4,6,4} \leq 1, x_{5,6,4} \leq 1, x_{6,6,4} \\
&\leq 1, x_{7,6,4} \leq 0, x_{8,6,4} \leq 0, x_{1,7,4} \leq 1, x_{2,7,4} \leq 1, x_{3,7,4} \leq 1, x_{4,7,4} \leq 1, x_{5,7,4} \\
&\leq 1, x_{6,7,4} \leq 1, x_{7,7,4} \leq 0, x_{8,7,4} \leq 0, x_{1,8,4} \leq 1, x_{2,8,4} \leq 1, x_{3,8,4} \leq 1, x_{4,8,4} \\
&\leq 1, x_{5,8,4} \leq 1, x_{6,8,4} \leq 1, x_{7,8,4} \leq 0, x_{8,8,4} \leq 0, x_{1,1,5} \leq 1, x_{2,1,5} \leq 1, x_{3,1,5} \\
&\leq 1, x_{4,1,5} \leq 1, x_{5,1,5} \leq 1, x_{6,1,5} \leq 1, x_{7,1,5} \leq 1, x_{8,1,5} \leq 1, x_{1,2,5} \leq 1, x_{2,2,5} \\
&\leq 1, x_{3,2,5} \leq 1, x_{4,2,5} \leq 1, x_{5,2,5} \leq 1, x_{6,2,5} \leq 1, x_{7,2,5} \leq 1, x_{8,2,5} \leq 1, x_{1,3,5} \\
&\leq 1, x_{2,3,5} \leq 1, x_{3,3,5} \leq 1, x_{4,3,5} \leq 1, x_{5,3,5} \leq 1, x_{6,3,5} \leq 1, x_{7,3,5} \leq 1, x_{8,3,5} \\
&\leq 1, x_{1,4,5} \leq 1, x_{2,4,5} \leq 1, x_{3,4,5} \leq 1, x_{4,4,5} \leq 1, x_{5,4,5} \leq 1, x_{6,4,5} \leq 1, x_{7,4,5} \\
&\leq 1, x_{8,4,5} \leq 1, x_{1,5,5} \leq 1, x_{2,5,5} \leq 1, x_{3,5,5} \leq 1, x_{4,5,5} \leq 1, x_{5,5,5} \leq 1, x_{6,5,5} \\
&\leq 1, x_{7,5,5} \leq 1, x_{8,5,5} \leq 1, x_{1,6,5} \leq 1, x_{2,6,5} \leq 1, x_{3,6,5} \leq 1, x_{4,6,5} \leq 1, x_{5,6,5} \\
&\leq 1, x_{6,6,5} \leq 1, x_{7,6,5} \leq 1, x_{8,6,5} \leq 1, x_{1,7,5} \leq 1, x_{2,7,5} \leq 1, x_{3,7,5} \leq 1, x_{4,7,5} \\
&\leq 1, x_{5,7,5} \leq 1, x_{6,7,5} \leq 1, x_{7,7,5} \leq 1, x_{8,7,5} \leq 1, x_{1,8,5} \leq 1, x_{2,8,5} \leq 1, x_{3,8,5} \\
&\leq 1, x_{4,8,5} \leq 1, x_{5,8,5} \leq 1, x_{6,8,5} \leq 1, x_{7,8,5} \leq 1, x_{8,8,5} \leq 1, x_{1,1,6} \leq 1, x_{2,1,6}
\end{aligned}$$

$$\begin{aligned}
&\leq 1, x_{3,1,6} \leq 1, x_{4,1,6} \leq 1, x_{5,1,6} \leq 1, x_{6,1,6} \leq 1, x_{7,1,6} \leq 1, x_{8,1,6} \leq 1, x_{1,2,6} \\
&\leq 1, x_{2,2,6} \leq 1, x_{3,2,6} \leq 1, x_{4,2,6} \leq 1, x_{5,2,6} \leq 1, x_{6,2,6} \leq 1, x_{7,2,6} \leq 1, x_{8,2,6} \\
&\leq 1, x_{1,3,6} \leq 1, x_{2,3,6} \leq 1, x_{3,3,6} \leq 1, x_{4,3,6} \leq 1, x_{5,3,6} \leq 1, x_{6,3,6} \leq 1, x_{7,3,6} \\
&\leq 1, x_{8,3,6} \leq 1, x_{1,4,6} \leq 1, x_{2,4,6} \leq 1, x_{3,4,6} \leq 1, x_{4,4,6} \leq 1, x_{5,4,6} \leq 1, x_{6,4,6} \\
&\leq 1, x_{7,4,6} \leq 1, x_{8,4,6} \leq 1, x_{1,5,6} \leq 1, x_{2,5,6} \leq 1, x_{3,5,6} \leq 1, x_{4,5,6} \leq 1, x_{5,5,6} \\
&\leq 1, x_{6,5,6} \leq 1, x_{7,5,6} \leq 1, x_{8,5,6} \leq 1, x_{1,6,6} \leq 1, x_{2,6,6} \leq 1, x_{3,6,6} \leq 1, x_{4,6,6} \\
&\leq 1, x_{5,6,6} \leq 1, x_{6,6,6} \leq 1, x_{7,6,6} \leq 1, x_{8,6,6} \leq 1, x_{1,7,6} \leq 1, x_{2,7,6} \leq 1, x_{3,7,6} \\
&\leq 1, x_{4,7,6} \leq 1, x_{5,7,6} \leq 1, x_{6,7,6} \leq 1, x_{7,7,6} \leq 1, x_{8,7,6} \leq 1, x_{1,8,6} \leq 1, x_{2,8,6} \\
&\leq 1, x_{3,8,6} \leq 1, x_{4,8,6} \leq 1, x_{5,8,6} \leq 1, x_{6,8,6} \leq 1, x_{7,8,6} \leq 1, x_{8,8,6} \leq 1, x_{1,1,7} \\
&\leq 1, x_{2,1,7} \leq 1, x_{3,1,7} \leq 1, x_{4,1,7} \leq 1, x_{5,1,7} \leq 1, x_{6,1,7} \leq 1, x_{7,1,7} \leq 1, x_{8,1,7} \\
&\leq 1, x_{1,2,7} \leq 1, x_{2,2,7} \leq 1, x_{3,2,7} \leq 1, x_{4,2,7} \leq 1, x_{5,2,7} \leq 1, x_{6,2,7} \leq 1, x_{7,2,7} \\
&\leq 1, x_{8,2,7} \leq 1, x_{1,3,7} \leq 1, x_{2,3,7} \leq 1, x_{3,3,7} \leq 1, x_{4,3,7} \leq 1, x_{5,3,7} \leq 1, x_{6,3,7} \\
&\leq 1, x_{7,3,7} \leq 1, x_{8,3,7} \leq 1, x_{1,4,7} \leq 1, x_{2,4,7} \leq 1, x_{3,4,7} \leq 1, x_{4,4,7} \leq 1, x_{5,4,7} \\
&\leq 1, x_{6,4,7} \leq 1, x_{7,4,7} \leq 1, x_{8,4,7} \leq 1, x_{1,5,7} \leq 1, x_{2,5,7} \leq 1, x_{3,5,7} \leq 1, x_{4,5,7} \\
&\leq 1, x_{5,5,7} \leq 1, x_{6,5,7} \leq 1, x_{7,5,7} \leq 1, x_{8,5,7} \leq 1, x_{1,6,7} \leq 1, x_{2,6,7} \leq 1, x_{3,6,7} \\
&\leq 1, x_{4,6,7} \leq 1, x_{5,6,7} \leq 1, x_{6,6,7} \leq 1, x_{7,6,7} \leq 1, x_{8,6,7} \leq 1, x_{1,7,7} \leq 1, x_{2,7,7} \\
&\leq 1, x_{3,7,7} \leq 1, x_{4,7,7} \leq 1, x_{5,7,7} \leq 1, x_{6,7,7} \leq 1, x_{7,7,7} \leq 1, x_{8,7,7} \leq 1, x_{1,8,7} \\
&\leq 1, x_{2,8,7} \leq 1, x_{3,8,7} \leq 1, x_{4,8,7} \leq 1, x_{5,8,7} \leq 1, x_{6,8,7} \leq 1, x_{7,8,7} \leq 1, x_{8,8,7} \\
&\leq 1, x_{1,1,8} \leq 1, x_{2,1,8} \leq 1, x_{3,1,8} \leq 0, x_{4,1,8} \leq 0, x_{5,1,8} \leq 1, x_{6,1,8} \leq 1, x_{7,1,8} \\
&\leq 1, x_{8,1,8} \leq 1, x_{1,2,8} \leq 1, x_{2,2,8} \leq 1, x_{3,2,8} \leq 0, x_{4,2,8} \leq 0, x_{5,2,8} \leq 1, x_{6,2,8} \\
&\leq 1, x_{7,2,8} \leq 1, x_{8,2,8} \leq 1, x_{1,3,8} \leq 1, x_{2,3,8} \leq 1, x_{3,3,8} \leq 0, x_{4,3,8} \leq 0, x_{5,3,8} \\
&\leq 1, x_{6,3,8} \leq 1, x_{7,3,8} \leq 1, x_{8,3,8} \leq 1, x_{1,4,8} \leq 1, x_{2,4,8} \leq 1, x_{3,4,8} \leq 0, x_{4,4,8} \\
&\leq 0, x_{5,4,8} \leq 1, x_{6,4,8} \leq 1, x_{7,4,8} \leq 1, x_{8,4,8} \leq 1, x_{1,5,8} \leq 1, x_{2,5,8} \leq 1, x_{3,5,8} \\
&\leq 0, x_{4,5,8} \leq 0, x_{5,5,8} \leq 1, x_{6,5,8} \leq 1, x_{7,5,8} \leq 1, x_{8,5,8} \leq 1, x_{1,6,8} \leq 1, x_{2,6,8} \\
&\leq 1, x_{3,6,8} \leq 0, x_{4,6,8} \leq 0, x_{5,6,8} \leq 1, x_{6,6,8} \leq 1, x_{7,6,8} \leq 1, x_{8,6,8} \leq 1, x_{1,7,8} \\
&\leq 1, x_{2,7,8} \leq 1, x_{3,7,8} \leq 0, x_{4,7,8} \leq 0, x_{5,7,8} \leq 1, x_{6,7,8} \leq 1, x_{7,7,8} \leq 1, x_{8,7,8} \\
&\leq 1, x_{1,8,8} \leq 1, x_{2,8,8} \leq 1, x_{3,8,8} \leq 0, x_{4,8,8} \leq 0, x_{5,8,8} \leq 1, x_{6,8,8} \leq 1, x_{7,8,8} \\
&\leq 1, x_{8,8,8} \leq 1
\end{aligned}$$

> **ByeWeeks2 := seq(seq(seq(x[j,i,k]<=abs(bMatrix[i,k]-1), i=1..n), j=1..n), k=1..m);**

$$\begin{aligned}
\text{ByeWeeks2} &:= x_{1,1,1} \leq 0, x_{1,2,1} \leq 0, x_{1,3,1} \leq 1, x_{1,4,1} \leq 1, x_{1,5,1} \leq 1, x_{1,6,1} \leq 1, x_{1,7,1} \\
&\leq 1, x_{1,8,1} \leq 1, x_{2,1,1} \leq 0, x_{2,2,1} \leq 0, x_{2,3,1} \leq 1, x_{2,4,1} \leq 1, x_{2,5,1} \leq 1, x_{2,6,1}
\end{aligned} \tag{5}$$

$$\begin{aligned}
&\leq 1, x_{2,7,1} \leq 1, x_{2,8,1} \leq 1, x_{3,1,1} \leq 0, x_{3,2,1} \leq 0, x_{3,3,1} \leq 1, x_{3,4,1} \leq 1, x_{3,5,1} \\
&\leq 1, x_{3,6,1} \leq 1, x_{3,7,1} \leq 1, x_{3,8,1} \leq 1, x_{4,1,1} \leq 0, x_{4,2,1} \leq 0, x_{4,3,1} \leq 1, x_{4,4,1} \\
&\leq 1, x_{4,5,1} \leq 1, x_{4,6,1} \leq 1, x_{4,7,1} \leq 1, x_{4,8,1} \leq 1, x_{5,1,1} \leq 0, x_{5,2,1} \leq 0, x_{5,3,1} \\
&\leq 1, x_{5,4,1} \leq 1, x_{5,5,1} \leq 1, x_{5,6,1} \leq 1, x_{5,7,1} \leq 1, x_{5,8,1} \leq 1, x_{6,1,1} \leq 0, x_{6,2,1} \\
&\leq 0, x_{6,3,1} \leq 1, x_{6,4,1} \leq 1, x_{6,5,1} \leq 1, x_{6,6,1} \leq 1, x_{6,7,1} \leq 1, x_{6,8,1} \leq 1, x_{7,1,1} \\
&\leq 0, x_{7,2,1} \leq 0, x_{7,3,1} \leq 1, x_{7,4,1} \leq 1, x_{7,5,1} \leq 1, x_{7,6,1} \leq 1, x_{7,7,1} \leq 1, x_{7,8,1} \\
&\leq 1, x_{8,1,1} \leq 0, x_{8,2,1} \leq 0, x_{8,3,1} \leq 1, x_{8,4,1} \leq 1, x_{8,5,1} \leq 1, x_{8,6,1} \leq 1, x_{8,7,1} \\
&\leq 1, x_{8,8,1} \leq 1, x_{1,1,2} \leq 1, x_{1,2,2} \leq 1, x_{1,3,2} \leq 1, x_{1,4,2} \leq 1, x_{1,5,2} \leq 1, x_{1,6,2} \\
&\leq 1, x_{1,7,2} \leq 1, x_{1,8,2} \leq 1, x_{2,1,2} \leq 1, x_{2,2,2} \leq 1, x_{2,3,2} \leq 1, x_{2,4,2} \leq 1, x_{2,5,2} \\
&\leq 1, x_{2,6,2} \leq 1, x_{2,7,2} \leq 1, x_{2,8,2} \leq 1, x_{3,1,2} \leq 1, x_{3,2,2} \leq 1, x_{3,3,2} \leq 1, x_{3,4,2} \\
&\leq 1, x_{3,5,2} \leq 1, x_{3,6,2} \leq 1, x_{3,7,2} \leq 1, x_{3,8,2} \leq 1, x_{4,1,2} \leq 1, x_{4,2,2} \leq 1, x_{4,3,2} \\
&\leq 1, x_{4,4,2} \leq 1, x_{4,5,2} \leq 1, x_{4,6,2} \leq 1, x_{4,7,2} \leq 1, x_{4,8,2} \leq 1, x_{5,1,2} \leq 1, x_{5,2,2} \\
&\leq 1, x_{5,3,2} \leq 1, x_{5,4,2} \leq 1, x_{5,5,2} \leq 1, x_{5,6,2} \leq 1, x_{5,7,2} \leq 1, x_{5,8,2} \leq 1, x_{6,1,2} \\
&\leq 1, x_{6,2,2} \leq 1, x_{6,3,2} \leq 1, x_{6,4,2} \leq 1, x_{6,5,2} \leq 1, x_{6,6,2} \leq 1, x_{6,7,2} \leq 1, x_{6,8,2} \\
&\leq 1, x_{7,1,2} \leq 1, x_{7,2,2} \leq 1, x_{7,3,2} \leq 1, x_{7,4,2} \leq 1, x_{7,5,2} \leq 1, x_{7,6,2} \leq 1, x_{7,7,2} \\
&\leq 1, x_{7,8,2} \leq 1, x_{8,1,2} \leq 1, x_{8,2,2} \leq 1, x_{8,3,2} \leq 1, x_{8,4,2} \leq 1, x_{8,5,2} \leq 1, x_{8,6,2} \\
&\leq 1, x_{8,7,2} \leq 1, x_{8,8,2} \leq 1, x_{1,1,3} \leq 1, x_{1,2,3} \leq 1, x_{1,3,3} \leq 1, x_{1,4,3} \leq 1, x_{1,5,3} \\
&\leq 0, x_{1,6,3} \leq 0, x_{1,7,3} \leq 1, x_{1,8,3} \leq 1, x_{2,1,3} \leq 1, x_{2,2,3} \leq 1, x_{2,3,3} \leq 1, x_{2,4,3} \\
&\leq 1, x_{2,5,3} \leq 0, x_{2,6,3} \leq 0, x_{2,7,3} \leq 1, x_{2,8,3} \leq 1, x_{3,1,3} \leq 1, x_{3,2,3} \leq 1, x_{3,3,3} \\
&\leq 1, x_{3,4,3} \leq 1, x_{3,5,3} \leq 0, x_{3,6,3} \leq 0, x_{3,7,3} \leq 1, x_{3,8,3} \leq 1, x_{4,1,3} \leq 1, x_{4,2,3} \\
&\leq 1, x_{4,3,3} \leq 1, x_{4,4,3} \leq 1, x_{4,5,3} \leq 0, x_{4,6,3} \leq 0, x_{4,7,3} \leq 1, x_{4,8,3} \leq 1, x_{5,1,3} \\
&\leq 1, x_{5,2,3} \leq 1, x_{5,3,3} \leq 1, x_{5,4,3} \leq 1, x_{5,5,3} \leq 0, x_{5,6,3} \leq 0, x_{5,7,3} \leq 1, x_{5,8,3} \\
&\leq 1, x_{6,1,3} \leq 1, x_{6,2,3} \leq 1, x_{6,3,3} \leq 1, x_{6,4,3} \leq 1, x_{6,5,3} \leq 0, x_{6,6,3} \leq 0, x_{6,7,3} \\
&\leq 1, x_{6,8,3} \leq 1, x_{7,1,3} \leq 1, x_{7,2,3} \leq 1, x_{7,3,3} \leq 1, x_{7,4,3} \leq 1, x_{7,5,3} \leq 0, x_{7,6,3} \\
&\leq 0, x_{7,7,3} \leq 1, x_{7,8,3} \leq 1, x_{8,1,3} \leq 1, x_{8,2,3} \leq 1, x_{8,3,3} \leq 1, x_{8,4,3} \leq 1, x_{8,5,3} \\
&\leq 0, x_{8,6,3} \leq 0, x_{8,7,3} \leq 1, x_{8,8,3} \leq 1, x_{1,1,4} \leq 1, x_{1,2,4} \leq 1, x_{1,3,4} \leq 1, x_{1,4,4} \\
&\leq 1, x_{1,5,4} \leq 1, x_{1,6,4} \leq 1, x_{1,7,4} \leq 0, x_{1,8,4} \leq 0, x_{2,1,4} \leq 1, x_{2,2,4} \leq 1, x_{2,3,4} \\
&\leq 1, x_{2,4,4} \leq 1, x_{2,5,4} \leq 1, x_{2,6,4} \leq 1, x_{2,7,4} \leq 0, x_{2,8,4} \leq 0, x_{3,1,4} \leq 1, x_{3,2,4} \\
&\leq 1, x_{3,3,4} \leq 1, x_{3,4,4} \leq 1, x_{3,5,4} \leq 1, x_{3,6,4} \leq 1, x_{3,7,4} \leq 0, x_{3,8,4} \leq 0, x_{4,1,4} \\
&\leq 1, x_{4,2,4} \leq 1, x_{4,3,4} \leq 1, x_{4,4,4} \leq 1, x_{4,5,4} \leq 1, x_{4,6,4} \leq 1, x_{4,7,4} \leq 0, x_{4,8,4} \\
&\leq 0, x_{5,1,4} \leq 1, x_{5,2,4} \leq 1, x_{5,3,4} \leq 1, x_{5,4,4} \leq 1, x_{5,5,4} \leq 1, x_{5,6,4} \leq 1, x_{5,7,4}
\end{aligned}$$

$$\begin{aligned}
&\leq 0, x_{5,8,4} \leq 0, x_{6,1,4} \leq 1, x_{6,2,4} \leq 1, x_{6,3,4} \leq 1, x_{6,4,4} \leq 1, x_{6,5,4} \leq 1, x_{6,6,4} \\
&\leq 1, x_{6,7,4} \leq 0, x_{6,8,4} \leq 0, x_{7,1,4} \leq 1, x_{7,2,4} \leq 1, x_{7,3,4} \leq 1, x_{7,4,4} \leq 1, x_{7,5,4} \\
&\leq 1, x_{7,6,4} \leq 1, x_{7,7,4} \leq 0, x_{7,8,4} \leq 0, x_{8,1,4} \leq 1, x_{8,2,4} \leq 1, x_{8,3,4} \leq 1, x_{8,4,4} \\
&\leq 1, x_{8,5,4} \leq 1, x_{8,6,4} \leq 1, x_{8,7,4} \leq 0, x_{8,8,4} \leq 0, x_{1,1,5} \leq 1, x_{1,2,5} \leq 1, x_{1,3,5} \\
&\leq 1, x_{1,4,5} \leq 1, x_{1,5,5} \leq 1, x_{1,6,5} \leq 1, x_{1,7,5} \leq 1, x_{1,8,5} \leq 1, x_{2,1,5} \leq 1, x_{2,2,5} \\
&\leq 1, x_{2,3,5} \leq 1, x_{2,4,5} \leq 1, x_{2,5,5} \leq 1, x_{2,6,5} \leq 1, x_{2,7,5} \leq 1, x_{2,8,5} \leq 1, x_{3,1,5} \\
&\leq 1, x_{3,2,5} \leq 1, x_{3,3,5} \leq 1, x_{3,4,5} \leq 1, x_{3,5,5} \leq 1, x_{3,6,5} \leq 1, x_{3,7,5} \leq 1, x_{3,8,5} \\
&\leq 1, x_{4,1,5} \leq 1, x_{4,2,5} \leq 1, x_{4,3,5} \leq 1, x_{4,4,5} \leq 1, x_{4,5,5} \leq 1, x_{4,6,5} \leq 1, x_{4,7,5} \\
&\leq 1, x_{4,8,5} \leq 1, x_{5,1,5} \leq 1, x_{5,2,5} \leq 1, x_{5,3,5} \leq 1, x_{5,4,5} \leq 1, x_{5,5,5} \leq 1, x_{5,6,5} \\
&\leq 1, x_{5,7,5} \leq 1, x_{5,8,5} \leq 1, x_{6,1,5} \leq 1, x_{6,2,5} \leq 1, x_{6,3,5} \leq 1, x_{6,4,5} \leq 1, x_{6,5,5} \\
&\leq 1, x_{6,6,5} \leq 1, x_{6,7,5} \leq 1, x_{6,8,5} \leq 1, x_{7,1,5} \leq 1, x_{7,2,5} \leq 1, x_{7,3,5} \leq 1, x_{7,4,5} \\
&\leq 1, x_{7,5,5} \leq 1, x_{7,6,5} \leq 1, x_{7,7,5} \leq 1, x_{7,8,5} \leq 1, x_{8,1,5} \leq 1, x_{8,2,5} \leq 1, x_{8,3,5} \\
&\leq 1, x_{8,4,5} \leq 1, x_{8,5,5} \leq 1, x_{8,6,5} \leq 1, x_{8,7,5} \leq 1, x_{8,8,5} \leq 1, x_{1,1,6} \leq 1, x_{1,2,6} \\
&\leq 1, x_{1,3,6} \leq 1, x_{1,4,6} \leq 1, x_{1,5,6} \leq 1, x_{1,6,6} \leq 1, x_{1,7,6} \leq 1, x_{1,8,6} \leq 1, x_{2,1,6} \\
&\leq 1, x_{2,2,6} \leq 1, x_{2,3,6} \leq 1, x_{2,4,6} \leq 1, x_{2,5,6} \leq 1, x_{2,6,6} \leq 1, x_{2,7,6} \leq 1, x_{2,8,6} \\
&\leq 1, x_{3,1,6} \leq 1, x_{3,2,6} \leq 1, x_{3,3,6} \leq 1, x_{3,4,6} \leq 1, x_{3,5,6} \leq 1, x_{3,6,6} \leq 1, x_{3,7,6} \\
&\leq 1, x_{3,8,6} \leq 1, x_{4,1,6} \leq 1, x_{4,2,6} \leq 1, x_{4,3,6} \leq 1, x_{4,4,6} \leq 1, x_{4,5,6} \leq 1, x_{4,6,6} \\
&\leq 1, x_{4,7,6} \leq 1, x_{4,8,6} \leq 1, x_{5,1,6} \leq 1, x_{5,2,6} \leq 1, x_{5,3,6} \leq 1, x_{5,4,6} \leq 1, x_{5,5,6} \\
&\leq 1, x_{5,6,6} \leq 1, x_{5,7,6} \leq 1, x_{5,8,6} \leq 1, x_{6,1,6} \leq 1, x_{6,2,6} \leq 1, x_{6,3,6} \leq 1, x_{6,4,6} \\
&\leq 1, x_{6,5,6} \leq 1, x_{6,6,6} \leq 1, x_{6,7,6} \leq 1, x_{6,8,6} \leq 1, x_{7,1,6} \leq 1, x_{7,2,6} \leq 1, x_{7,3,6} \\
&\leq 1, x_{7,4,6} \leq 1, x_{7,5,6} \leq 1, x_{7,6,6} \leq 1, x_{7,7,6} \leq 1, x_{7,8,6} \leq 1, x_{8,1,6} \leq 1, x_{8,2,6} \\
&\leq 1, x_{8,3,6} \leq 1, x_{8,4,6} \leq 1, x_{8,5,6} \leq 1, x_{8,6,6} \leq 1, x_{8,7,6} \leq 1, x_{8,8,6} \leq 1, x_{1,1,7} \\
&\leq 1, x_{1,2,7} \leq 1, x_{1,3,7} \leq 1, x_{1,4,7} \leq 1, x_{1,5,7} \leq 1, x_{1,6,7} \leq 1, x_{1,7,7} \leq 1, x_{1,8,7} \\
&\leq 1, x_{2,1,7} \leq 1, x_{2,2,7} \leq 1, x_{2,3,7} \leq 1, x_{2,4,7} \leq 1, x_{2,5,7} \leq 1, x_{2,6,7} \leq 1, x_{2,7,7} \\
&\leq 1, x_{2,8,7} \leq 1, x_{3,1,7} \leq 1, x_{3,2,7} \leq 1, x_{3,3,7} \leq 1, x_{3,4,7} \leq 1, x_{3,5,7} \leq 1, x_{3,6,7} \\
&\leq 1, x_{3,7,7} \leq 1, x_{3,8,7} \leq 1, x_{4,1,7} \leq 1, x_{4,2,7} \leq 1, x_{4,3,7} \leq 1, x_{4,4,7} \leq 1, x_{4,5,7} \\
&\leq 1, x_{4,6,7} \leq 1, x_{4,7,7} \leq 1, x_{4,8,7} \leq 1, x_{5,1,7} \leq 1, x_{5,2,7} \leq 1, x_{5,3,7} \leq 1, x_{5,4,7} \\
&\leq 1, x_{5,5,7} \leq 1, x_{5,6,7} \leq 1, x_{5,7,7} \leq 1, x_{5,8,7} \leq 1, x_{6,1,7} \leq 1, x_{6,2,7} \leq 1, x_{6,3,7} \\
&\leq 1, x_{6,4,7} \leq 1, x_{6,5,7} \leq 1, x_{6,6,7} \leq 1, x_{6,7,7} \leq 1, x_{6,8,7} \leq 1, x_{7,1,7} \leq 1, x_{7,2,7} \\
&\leq 1, x_{7,3,7} \leq 1, x_{7,4,7} \leq 1, x_{7,5,7} \leq 1, x_{7,6,7} \leq 1, x_{7,7,7} \leq 1, x_{7,8,7} \leq 1, x_{8,1,7} \\
&\leq 1, x_{8,2,7} \leq 1, x_{8,3,7} \leq 1, x_{8,4,7} \leq 1, x_{8,5,7} \leq 1, x_{8,6,7} \leq 1, x_{8,7,7} \leq 1, x_{8,8,7}
\end{aligned}$$

$$\begin{aligned}
&\leq 1, x_{1,1,8} \leq 1, x_{1,2,8} \leq 1, x_{1,3,8} \leq 0, x_{1,4,8} \leq 0, x_{1,5,8} \leq 1, x_{1,6,8} \leq 1, x_{1,7,8} \\
&\leq 1, x_{1,8,8} \leq 1, x_{2,1,8} \leq 1, x_{2,2,8} \leq 1, x_{2,3,8} \leq 0, x_{2,4,8} \leq 0, x_{2,5,8} \leq 1, x_{2,6,8} \\
&\leq 1, x_{2,7,8} \leq 1, x_{2,8,8} \leq 1, x_{3,1,8} \leq 1, x_{3,2,8} \leq 1, x_{3,3,8} \leq 0, x_{3,4,8} \leq 0, x_{3,5,8} \\
&\leq 1, x_{3,6,8} \leq 1, x_{3,7,8} \leq 1, x_{3,8,8} \leq 1, x_{4,1,8} \leq 1, x_{4,2,8} \leq 1, x_{4,3,8} \leq 0, x_{4,4,8} \\
&\leq 0, x_{4,5,8} \leq 1, x_{4,6,8} \leq 1, x_{4,7,8} \leq 1, x_{4,8,8} \leq 1, x_{5,1,8} \leq 1, x_{5,2,8} \leq 1, x_{5,3,8} \\
&\leq 0, x_{5,4,8} \leq 0, x_{5,5,8} \leq 1, x_{5,6,8} \leq 1, x_{5,7,8} \leq 1, x_{5,8,8} \leq 1, x_{6,1,8} \leq 1, x_{6,2,8} \\
&\leq 1, x_{6,3,8} \leq 0, x_{6,4,8} \leq 0, x_{6,5,8} \leq 1, x_{6,6,8} \leq 1, x_{6,7,8} \leq 1, x_{6,8,8} \leq 1, x_{7,1,8} \\
&\leq 1, x_{7,2,8} \leq 1, x_{7,3,8} \leq 0, x_{7,4,8} \leq 0, x_{7,5,8} \leq 1, x_{7,6,8} \leq 1, x_{7,7,8} \leq 1, x_{7,8,8} \\
&\leq 1, x_{8,1,8} \leq 1, x_{8,2,8} \leq 1, x_{8,3,8} \leq 0, x_{8,4,8} \leq 0, x_{8,5,8} \leq 1, x_{8,6,8} \leq 1, x_{8,7,8} \\
&\leq 1, x_{8,8,8} \leq 1
\end{aligned}$$

> More := seq(seq(seq(x[i,j,k] = x[j,i,k], i=1..n), j=1..n), k=1..m);

More := $x_{1,1,1} = x_{1,1,1}, x_{2,1,1} = x_{1,2,1}, x_{3,1,1} = x_{1,3,1}, x_{4,1,1} = x_{1,4,1}, x_{5,1,1} = x_{1,5,1},$
 $x_{6,1,1} = x_{1,6,1}, x_{7,1,1} = x_{1,7,1}, x_{8,1,1} = x_{1,8,1}, x_{1,2,1} = x_{2,1,1}, x_{2,2,1} = x_{2,2,1}, x_{3,2,1}$
 $= x_{2,3,1}, x_{4,2,1} = x_{2,4,1}, x_{5,2,1} = x_{2,5,1}, x_{6,2,1} = x_{2,6,1}, x_{7,2,1} = x_{2,7,1}, x_{8,2,1} = x_{2,8,1},$
 $x_{1,3,1} = x_{3,1,1}, x_{2,3,1} = x_{3,2,1}, x_{3,3,1} = x_{3,3,1}, x_{4,3,1} = x_{3,4,1}, x_{5,3,1} = x_{3,5,1}, x_{6,3,1}$
 $= x_{3,6,1}, x_{7,3,1} = x_{3,7,1}, x_{8,3,1} = x_{3,8,1}, x_{1,4,1} = x_{4,1,1}, x_{2,4,1} = x_{4,2,1}, x_{3,4,1} = x_{4,3,1},$
 $x_{4,4,1} = x_{4,4,1}, x_{5,4,1} = x_{4,5,1}, x_{6,4,1} = x_{4,6,1}, x_{7,4,1} = x_{4,7,1}, x_{8,4,1} = x_{4,8,1}, x_{1,5,1}$
 $= x_{5,1,1}, x_{2,5,1} = x_{5,2,1}, x_{3,5,1} = x_{5,3,1}, x_{4,5,1} = x_{5,4,1}, x_{5,5,1} = x_{5,5,1}, x_{6,5,1} = x_{5,6,1},$
 $x_{7,5,1} = x_{5,7,1}, x_{8,5,1} = x_{5,8,1}, x_{1,6,1} = x_{6,1,1}, x_{2,6,1} = x_{6,2,1}, x_{3,6,1} = x_{6,3,1}, x_{4,6,1}$
 $= x_{6,4,1}, x_{5,6,1} = x_{6,5,1}, x_{6,6,1} = x_{6,6,1}, x_{7,6,1} = x_{6,7,1}, x_{8,6,1} = x_{6,8,1}, x_{1,7,1} = x_{7,1,1},$
 $x_{2,7,1} = x_{7,2,1}, x_{3,7,1} = x_{7,3,1}, x_{4,7,1} = x_{7,4,1}, x_{5,7,1} = x_{7,5,1}, x_{6,7,1} = x_{7,6,1}, x_{7,7,1}$
 $= x_{7,7,1}, x_{8,7,1} = x_{7,8,1}, x_{1,8,1} = x_{8,1,1}, x_{2,8,1} = x_{8,2,1}, x_{3,8,1} = x_{8,3,1}, x_{4,8,1} = x_{8,4,1},$
 $x_{5,8,1} = x_{8,5,1}, x_{6,8,1} = x_{8,6,1}, x_{7,8,1} = x_{8,7,1}, x_{8,8,1} = x_{8,8,1}, x_{1,1,2} = x_{1,1,2}, x_{2,1,2}$
 $= x_{1,2,2}, x_{3,1,2} = x_{1,3,2}, x_{4,1,2} = x_{1,4,2}, x_{5,1,2} = x_{1,5,2}, x_{6,1,2} = x_{1,6,2}, x_{7,1,2} = x_{1,7,2},$
 $x_{8,1,2} = x_{1,8,2}, x_{1,2,2} = x_{2,1,2}, x_{2,2,2} = x_{2,2,2}, x_{3,2,2} = x_{2,3,2}, x_{4,2,2} = x_{2,4,2}, x_{5,2,2}$
 $= x_{2,5,2}, x_{6,2,2} = x_{2,6,2}, x_{7,2,2} = x_{2,7,2}, x_{8,2,2} = x_{2,8,2}, x_{1,3,2} = x_{3,1,2}, x_{2,3,2} = x_{3,2,2},$
 $x_{3,3,2} = x_{3,3,2}, x_{4,3,2} = x_{3,4,2}, x_{5,3,2} = x_{3,5,2}, x_{6,3,2} = x_{3,6,2}, x_{7,3,2} = x_{3,7,2}, x_{8,3,2}$
 $= x_{3,8,2}, x_{1,4,2} = x_{4,1,2}, x_{2,4,2} = x_{4,2,2}, x_{3,4,2} = x_{4,3,2}, x_{4,4,2} = x_{4,4,2}, x_{5,4,2} = x_{4,5,2},$
 $x_{6,4,2} = x_{4,6,2}, x_{7,4,2} = x_{4,7,2}, x_{8,4,2} = x_{4,8,2}, x_{1,5,2} = x_{5,1,2}, x_{2,5,2} = x_{5,2,2}, x_{3,5,2}$
 $= x_{5,3,2}, x_{4,5,2} = x_{5,4,2}, x_{5,5,2} = x_{5,5,2}, x_{6,5,2} = x_{5,6,2}, x_{7,5,2} = x_{5,7,2}, x_{8,5,2} = x_{5,8,2},$
 $x_{1,6,2} = x_{6,1,2}, x_{2,6,2} = x_{6,2,2}, x_{3,6,2} = x_{6,3,2}, x_{4,6,2} = x_{6,4,2}, x_{5,6,2} = x_{6,5,2}, x_{6,6,2}$

(6)

$$\begin{aligned}
& x_{1,4,5} = x_{4,1,5}, x_{2,4,5} = x_{4,2,5}, x_{3,4,5} = x_{4,3,5}, x_{4,4,5} = x_{4,4,5}, x_{5,4,5} = x_{4,5,5}, x_{6,4,5} \\
& = x_{4,6,5}, x_{7,4,5} = x_{4,7,5}, x_{8,4,5} = x_{4,8,5}, x_{1,5,5} = x_{5,1,5}, x_{2,5,5} = x_{5,2,5}, x_{3,5,5} = x_{5,3,5}, \\
& x_{4,5,5} = x_{5,4,5}, x_{5,5,5} = x_{5,5,5}, x_{6,5,5} = x_{5,6,5}, x_{7,5,5} = x_{5,7,5}, x_{8,5,5} = x_{5,8,5}, x_{1,6,5} \\
& = x_{6,1,5}, x_{2,6,5} = x_{6,2,5}, x_{3,6,5} = x_{6,3,5}, x_{4,6,5} = x_{6,4,5}, x_{5,6,5} = x_{6,5,5}, x_{6,6,5} = x_{6,6,5}, \\
& x_{7,6,5} = x_{6,7,5}, x_{8,6,5} = x_{6,8,5}, x_{1,7,5} = x_{7,1,5}, x_{2,7,5} = x_{7,2,5}, x_{3,7,5} = x_{7,3,5}, x_{4,7,5} \\
& = x_{7,4,5}, x_{5,7,5} = x_{7,5,5}, x_{6,7,5} = x_{7,6,5}, x_{7,7,5} = x_{7,7,5}, x_{8,7,5} = x_{7,8,5}, x_{1,8,5} = x_{8,1,5}, \\
& x_{2,8,5} = x_{8,2,5}, x_{3,8,5} = x_{8,3,5}, x_{4,8,5} = x_{8,4,5}, x_{5,8,5} = x_{8,5,5}, x_{6,8,5} = x_{8,6,5}, x_{7,8,5} \\
& = x_{8,7,5}, x_{8,8,5} = x_{8,8,5}, x_{1,1,6} = x_{1,1,6}, x_{2,1,6} = x_{1,2,6}, x_{3,1,6} = x_{1,3,6}, x_{4,1,6} = x_{1,4,6}, \\
& x_{5,1,6} = x_{1,5,6}, x_{6,1,6} = x_{1,6,6}, x_{7,1,6} = x_{1,7,6}, x_{8,1,6} = x_{1,8,6}, x_{1,2,6} = x_{2,1,6}, x_{2,2,6} \\
& = x_{2,2,6}, x_{3,2,6} = x_{2,3,6}, x_{4,2,6} = x_{2,4,6}, x_{5,2,6} = x_{2,5,6}, x_{6,2,6} = x_{2,6,6}, x_{7,2,6} = x_{2,7,6}, \\
& x_{8,2,6} = x_{2,8,6}, x_{1,3,6} = x_{3,1,6}, x_{2,3,6} = x_{3,2,6}, x_{3,3,6} = x_{3,3,6}, x_{4,3,6} = x_{3,4,6}, x_{5,3,6} \\
& = x_{3,5,6}, x_{6,3,6} = x_{3,6,6}, x_{7,3,6} = x_{3,7,6}, x_{8,3,6} = x_{3,8,6}, x_{1,4,6} = x_{4,1,6}, x_{2,4,6} = x_{4,2,6}, \\
& x_{3,4,6} = x_{4,3,6}, x_{4,4,6} = x_{4,4,6}, x_{5,4,6} = x_{4,5,6}, x_{6,4,6} = x_{4,6,6}, x_{7,4,6} = x_{4,7,6}, x_{8,4,6} \\
& = x_{4,8,6}, x_{1,5,6} = x_{5,1,6}, x_{2,5,6} = x_{5,2,6}, x_{3,5,6} = x_{5,3,6}, x_{4,5,6} = x_{5,4,6}, x_{5,5,6} = x_{5,5,6}, \\
& x_{6,5,6} = x_{5,6,6}, x_{7,5,6} = x_{5,7,6}, x_{8,5,6} = x_{5,8,6}, x_{1,6,6} = x_{6,1,6}, x_{2,6,6} = x_{6,2,6}, x_{3,6,6} \\
& = x_{6,3,6}, x_{4,6,6} = x_{6,4,6}, x_{5,6,6} = x_{6,5,6}, x_{6,6,6} = x_{6,6,6}, x_{7,6,6} = x_{6,7,6}, x_{8,6,6} = x_{6,8,6}, \\
& x_{1,7,6} = x_{7,1,6}, x_{2,7,6} = x_{7,2,6}, x_{3,7,6} = x_{7,3,6}, x_{4,7,6} = x_{7,4,6}, x_{5,7,6} = x_{7,5,6}, x_{6,7,6} \\
& = x_{7,6,6}, x_{7,7,6} = x_{7,7,6}, x_{8,7,6} = x_{7,8,6}, x_{1,8,6} = x_{8,1,6}, x_{2,8,6} = x_{8,2,6}, x_{3,8,6} = x_{8,3,6}, \\
& x_{4,8,6} = x_{8,4,6}, x_{5,8,6} = x_{8,5,6}, x_{6,8,6} = x_{8,6,6}, x_{7,8,6} = x_{8,7,6}, x_{8,8,6} = x_{8,8,6}, x_{1,1,7} \\
& = x_{1,1,7}, x_{2,1,7} = x_{1,2,7}, x_{3,1,7} = x_{1,3,7}, x_{4,1,7} = x_{1,4,7}, x_{5,1,7} = x_{1,5,7}, x_{6,1,7} = x_{1,6,7}, \\
& x_{7,1,7} = x_{1,7,7}, x_{8,1,7} = x_{1,8,7}, x_{1,2,7} = x_{2,1,7}, x_{2,2,7} = x_{2,2,7}, x_{3,2,7} = x_{2,3,7}, x_{4,2,7} \\
& = x_{2,4,7}, x_{5,2,7} = x_{2,5,7}, x_{6,2,7} = x_{2,6,7}, x_{7,2,7} = x_{2,7,7}, x_{8,2,7} = x_{2,8,7}, x_{1,3,7} = x_{3,1,7}, \\
& x_{2,3,7} = x_{3,2,7}, x_{3,3,7} = x_{3,3,7}, x_{4,3,7} = x_{3,4,7}, x_{5,3,7} = x_{3,5,7}, x_{6,3,7} = x_{3,6,7}, x_{7,3,7} \\
& = x_{3,7,7}, x_{8,3,7} = x_{3,8,7}, x_{1,4,7} = x_{4,1,7}, x_{2,4,7} = x_{4,2,7}, x_{3,4,7} = x_{4,3,7}, x_{4,4,7} = x_{4,4,7}, \\
& x_{5,4,7} = x_{4,5,7}, x_{6,4,7} = x_{4,6,7}, x_{7,4,7} = x_{4,7,7}, x_{8,4,7} = x_{4,8,7}, x_{1,5,7} = x_{5,1,7}, x_{2,5,7} \\
& = x_{5,2,7}, x_{3,5,7} = x_{5,3,7}, x_{4,5,7} = x_{5,4,7}, x_{5,5,7} = x_{5,5,7}, x_{6,5,7} = x_{5,6,7}, x_{7,5,7} = x_{5,7,7}, \\
& x_{8,5,7} = x_{5,8,7}, x_{1,6,7} = x_{6,1,7}, x_{2,6,7} = x_{6,2,7}, x_{3,6,7} = x_{6,3,7}, x_{4,6,7} = x_{6,4,7}, x_{5,6,7} \\
& = x_{6,5,7}, x_{6,6,7} = x_{6,6,7}, x_{7,6,7} = x_{6,7,7}, x_{8,6,7} = x_{6,8,7}, x_{1,7,7} = x_{7,1,7}, x_{2,7,7} = x_{7,2,7}, \\
& x_{3,7,7} = x_{7,3,7}, x_{4,7,7} = x_{7,4,7}, x_{5,7,7} = x_{7,5,7}, x_{6,7,7} = x_{7,6,7}, x_{7,7,7} = x_{7,7,7}, x_{8,7,7} \\
& = x_{7,8,7}, x_{1,8,7} = x_{8,1,7}, x_{2,8,7} = x_{8,2,7}, x_{3,8,7} = x_{8,3,7}, x_{4,8,7} = x_{8,4,7}, x_{5,8,7} = x_{8,5,7}, \\
& x_{6,8,7} = x_{8,6,7}, x_{7,8,7} = x_{8,7,7}, x_{8,8,7} = x_{8,8,7}, x_{1,1,8} = x_{1,1,8}, x_{2,1,8} = x_{1,2,8}, x_{3,1,8}
\end{aligned}$$

$$\begin{aligned}
&= x_{1,3,8} x_{4,1,8} = x_{1,4,8} x_{5,1,8} = x_{1,5,8} x_{6,1,8} = x_{1,6,8} x_{7,1,8} = x_{1,7,8} x_{8,1,8} = x_{1,8,8} x_{1,2,8} \\
&= x_{2,1,8} x_{2,2,8} = x_{2,2,8} x_{3,2,8} = x_{2,3,8} x_{4,2,8} = x_{2,4,8} x_{5,2,8} = x_{2,5,8} x_{6,2,8} \\
&= x_{2,6,8} x_{7,2,8} = x_{2,7,8} x_{8,2,8} = x_{2,8,8} x_{1,3,8} = x_{3,1,8} x_{2,3,8} = x_{3,2,8} x_{3,3,8} = x_{3,3,8} x_{4,3,8} \\
&= x_{3,4,8} x_{5,3,8} = x_{3,5,8} x_{6,3,8} = x_{3,6,8} x_{7,3,8} = x_{3,7,8} x_{8,3,8} = x_{3,8,8} x_{1,4,8} \\
&= x_{4,1,8} x_{2,4,8} = x_{4,2,8} x_{3,4,8} = x_{4,3,8} x_{4,4,8} = x_{4,4,8} x_{5,4,8} = x_{4,5,8} x_{6,4,8} = x_{4,6,8} x_{7,4,8} \\
&= x_{4,7,8} x_{8,4,8} = x_{4,8,8} x_{1,5,8} = x_{5,1,8} x_{2,5,8} = x_{5,2,8} x_{3,5,8} = x_{5,3,8} x_{4,5,8} \\
&= x_{5,4,8} x_{5,5,8} = x_{5,5,8} x_{6,5,8} = x_{5,6,8} x_{7,5,8} = x_{5,7,8} x_{8,5,8} = x_{5,8,8} x_{1,6,8} = x_{6,1,8} x_{2,6,8} \\
&= x_{6,2,8} x_{3,6,8} = x_{6,3,8} x_{4,6,8} = x_{6,4,8} x_{5,6,8} = x_{6,5,8} x_{6,6,8} = x_{6,6,8} x_{7,6,8} \\
&= x_{6,7,8} x_{8,6,8} = x_{6,8,8} x_{1,7,8} = x_{7,1,8} x_{2,7,8} = x_{7,2,8} x_{3,7,8} = x_{7,3,8} x_{4,7,8} = x_{7,4,8} x_{5,7,8} \\
&= x_{7,5,8} x_{6,7,8} = x_{7,6,8} x_{7,7,8} = x_{7,7,8} x_{8,7,8} = x_{7,8,8} x_{1,8,8} = x_{8,1,8} x_{2,8,8} \\
&= x_{8,2,8} x_{3,8,8} = x_{8,3,8} x_{4,8,8} = x_{8,4,8} x_{5,8,8} = x_{8,5,8} x_{6,8,8} = x_{8,6,8} x_{7,8,8} = x_{8,7,8} x_{8,8,8} \\
&= x_{8,8,8}
\end{aligned}$$

> **HTConstraints := seq(seq(add(x[i,j,k], i=1..n) <= 1, j=1..n), k=1..m);**

$$\begin{aligned}
&HTConstraints := x_{1,1,1} + x_{2,1,1} + x_{3,1,1} + x_{4,1,1} + x_{5,1,1} + x_{6,1,1} + x_{7,1,1} + x_{8,1,1} \\
&\leq 1, x_{1,2,1} + x_{2,2,1} + x_{3,2,1} + x_{4,2,1} + x_{5,2,1} + x_{6,2,1} + x_{7,2,1} + x_{8,2,1} \leq 1, x_{1,3,1} \\
&+ x_{2,3,1} + x_{3,3,1} + x_{4,3,1} + x_{5,3,1} + x_{6,3,1} + x_{7,3,1} + x_{8,3,1} \leq 1, x_{1,4,1} + x_{2,4,1} \\
&+ x_{3,4,1} + x_{4,4,1} + x_{5,4,1} + x_{6,4,1} + x_{7,4,1} + x_{8,4,1} \leq 1, x_{1,5,1} + x_{2,5,1} + x_{3,5,1} \\
&+ x_{4,5,1} + x_{5,5,1} + x_{6,5,1} + x_{7,5,1} + x_{8,5,1} \leq 1, x_{1,6,1} + x_{2,6,1} + x_{3,6,1} + x_{4,6,1} \\
&+ x_{5,6,1} + x_{6,6,1} + x_{7,6,1} + x_{8,6,1} \leq 1, x_{1,7,1} + x_{2,7,1} + x_{3,7,1} + x_{4,7,1} + x_{5,7,1} \\
&+ x_{6,7,1} + x_{7,7,1} + x_{8,7,1} \leq 1, x_{1,8,1} + x_{2,8,1} + x_{3,8,1} + x_{4,8,1} + x_{5,8,1} + x_{6,8,1} \\
&+ x_{7,8,1} + x_{8,8,1} \leq 1, x_{1,1,2} + x_{2,1,2} + x_{3,1,2} + x_{4,1,2} + x_{5,1,2} + x_{6,1,2} + x_{7,1,2} \\
&+ x_{8,1,2} \leq 1, x_{1,2,2} + x_{2,2,2} + x_{3,2,2} + x_{4,2,2} + x_{5,2,2} + x_{6,2,2} + x_{7,2,2} + x_{8,2,2} \\
&\leq 1, x_{1,3,2} + x_{2,3,2} + x_{3,3,2} + x_{4,3,2} + x_{5,3,2} + x_{6,3,2} + x_{7,3,2} + x_{8,3,2} \leq 1, x_{1,4,2} \\
&+ x_{2,4,2} + x_{3,4,2} + x_{4,4,2} + x_{5,4,2} + x_{6,4,2} + x_{7,4,2} + x_{8,4,2} \leq 1, x_{1,5,2} + x_{2,5,2} \\
&+ x_{3,5,2} + x_{4,5,2} + x_{5,5,2} + x_{6,5,2} + x_{7,5,2} + x_{8,5,2} \leq 1, x_{1,6,2} + x_{2,6,2} + x_{3,6,2} \\
&+ x_{4,6,2} + x_{5,6,2} + x_{6,6,2} + x_{7,6,2} + x_{8,6,2} \leq 1, x_{1,7,2} + x_{2,7,2} + x_{3,7,2} + x_{4,7,2} \\
&+ x_{5,7,2} + x_{6,7,2} + x_{7,7,2} + x_{8,7,2} \leq 1, x_{1,8,2} + x_{2,8,2} + x_{3,8,2} + x_{4,8,2} + x_{5,8,2} \\
&+ x_{6,8,2} + x_{7,8,2} + x_{8,8,2} \leq 1, x_{1,1,3} + x_{2,1,3} + x_{3,1,3} + x_{4,1,3} + x_{5,1,3} + x_{6,1,3} \\
&+ x_{7,1,3} + x_{8,1,3} \leq 1, x_{1,2,3} + x_{2,2,3} + x_{3,2,3} + x_{4,2,3} + x_{5,2,3} + x_{6,2,3} + x_{7,2,3} \\
&+ x_{8,2,3} \leq 1, x_{1,3,3} + x_{2,3,3} + x_{3,3,3} + x_{4,3,3} + x_{5,3,3} + x_{6,3,3} + x_{7,3,3} + x_{8,3,3} \\
&\leq 1, x_{1,4,3} + x_{2,4,3} + x_{3,4,3} + x_{4,4,3} + x_{5,4,3} + x_{6,4,3} + x_{7,4,3} + x_{8,4,3} \leq 1, x_{1,5,3}
\end{aligned}$$

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$$\begin{aligned}
& + x_{2,5,3} + x_{3,5,3} + x_{4,5,3} + x_{5,5,3} + x_{6,5,3} + x_{7,5,3} + x_{8,5,3} \leq 1, x_{1,6,3} + x_{2,6,3} \\
& + x_{3,6,3} + x_{4,6,3} + x_{5,6,3} + x_{6,6,3} + x_{7,6,3} + x_{8,6,3} \leq 1, x_{1,7,3} + x_{2,7,3} + x_{3,7,3} \\
& + x_{4,7,3} + x_{5,7,3} + x_{6,7,3} + x_{7,7,3} + x_{8,7,3} \leq 1, x_{1,8,3} + x_{2,8,3} + x_{3,8,3} + x_{4,8,3} \\
& + x_{5,8,3} + x_{6,8,3} + x_{7,8,3} + x_{8,8,3} \leq 1, x_{1,1,4} + x_{2,1,4} + x_{3,1,4} + x_{4,1,4} + x_{5,1,4} \\
& + x_{6,1,4} + x_{7,1,4} + x_{8,1,4} \leq 1, x_{1,2,4} + x_{2,2,4} + x_{3,2,4} + x_{4,2,4} + x_{5,2,4} + x_{6,2,4} \\
& + x_{7,2,4} + x_{8,2,4} \leq 1, x_{1,3,4} + x_{2,3,4} + x_{3,3,4} + x_{4,3,4} + x_{5,3,4} + x_{6,3,4} + x_{7,3,4} \\
& + x_{8,3,4} \leq 1, x_{1,4,4} + x_{2,4,4} + x_{3,4,4} + x_{4,4,4} + x_{5,4,4} + x_{6,4,4} + x_{7,4,4} + x_{8,4,4} \\
& \leq 1, x_{1,5,4} + x_{2,5,4} + x_{3,5,4} + x_{4,5,4} + x_{5,5,4} + x_{6,5,4} + x_{7,5,4} + x_{8,5,4} \leq 1, x_{1,6,4} \\
& + x_{2,6,4} + x_{3,6,4} + x_{4,6,4} + x_{5,6,4} + x_{6,6,4} + x_{7,6,4} + x_{8,6,4} \leq 1, x_{1,7,4} + x_{2,7,4} \\
& + x_{3,7,4} + x_{4,7,4} + x_{5,7,4} + x_{6,7,4} + x_{7,7,4} + x_{8,7,4} \leq 1, x_{1,8,4} + x_{2,8,4} + x_{3,8,4} \\
& + x_{4,8,4} + x_{5,8,4} + x_{6,8,4} + x_{7,8,4} + x_{8,8,4} \leq 1, x_{1,1,5} + x_{2,1,5} + x_{3,1,5} + x_{4,1,5} \\
& + x_{5,1,5} + x_{6,1,5} + x_{7,1,5} + x_{8,1,5} \leq 1, x_{1,2,5} + x_{2,2,5} + x_{3,2,5} + x_{4,2,5} + x_{5,2,5} \\
& + x_{6,2,5} + x_{7,2,5} + x_{8,2,5} \leq 1, x_{1,3,5} + x_{2,3,5} + x_{3,3,5} + x_{4,3,5} + x_{5,3,5} + x_{6,3,5} \\
& + x_{7,3,5} + x_{8,3,5} \leq 1, x_{1,4,5} + x_{2,4,5} + x_{3,4,5} + x_{4,4,5} + x_{5,4,5} + x_{6,4,5} + x_{7,4,5} \\
& + x_{8,4,5} \leq 1, x_{1,5,5} + x_{2,5,5} + x_{3,5,5} + x_{4,5,5} + x_{5,5,5} + x_{6,5,5} + x_{7,5,5} + x_{8,5,5} \\
& \leq 1, x_{1,6,5} + x_{2,6,5} + x_{3,6,5} + x_{4,6,5} + x_{5,6,5} + x_{6,6,5} + x_{7,6,5} + x_{8,6,5} \leq 1, x_{1,7,5} \\
& + x_{2,7,5} + x_{3,7,5} + x_{4,7,5} + x_{5,7,5} + x_{6,7,5} + x_{7,7,5} + x_{8,7,5} \leq 1, x_{1,8,5} + x_{2,8,5} \\
& + x_{3,8,5} + x_{4,8,5} + x_{5,8,5} + x_{6,8,5} + x_{7,8,5} + x_{8,8,5} \leq 1, x_{1,1,6} + x_{2,1,6} + x_{3,1,6} \\
& + x_{4,1,6} + x_{5,1,6} + x_{6,1,6} + x_{7,1,6} + x_{8,1,6} \leq 1, x_{1,2,6} + x_{2,2,6} + x_{3,2,6} + x_{4,2,6} \\
& + x_{5,2,6} + x_{6,2,6} + x_{7,2,6} + x_{8,2,6} \leq 1, x_{1,3,6} + x_{2,3,6} + x_{3,3,6} + x_{4,3,6} + x_{5,3,6} \\
& + x_{6,3,6} + x_{7,3,6} + x_{8,3,6} \leq 1, x_{1,4,6} + x_{2,4,6} + x_{3,4,6} + x_{4,4,6} + x_{5,4,6} + x_{6,4,6} \\
& + x_{7,4,6} + x_{8,4,6} \leq 1, x_{1,5,6} + x_{2,5,6} + x_{3,5,6} + x_{4,5,6} + x_{5,5,6} + x_{6,5,6} + x_{7,5,6} \\
& + x_{8,5,6} \leq 1, x_{1,6,6} + x_{2,6,6} + x_{3,6,6} + x_{4,6,6} + x_{5,6,6} + x_{6,6,6} + x_{7,6,6} + x_{8,6,6} \\
& \leq 1, x_{1,7,6} + x_{2,7,6} + x_{3,7,6} + x_{4,7,6} + x_{5,7,6} + x_{6,7,6} + x_{7,7,6} + x_{8,7,6} \leq 1, x_{1,8,6} \\
& + x_{2,8,6} + x_{3,8,6} + x_{4,8,6} + x_{5,8,6} + x_{6,8,6} + x_{7,8,6} + x_{8,8,6} \leq 1, x_{1,1,7} + x_{2,1,7} \\
& + x_{3,1,7} + x_{4,1,7} + x_{5,1,7} + x_{6,1,7} + x_{7,1,7} + x_{8,1,7} \leq 1, x_{1,2,7} + x_{2,2,7} + x_{3,2,7} \\
& + x_{4,2,7} + x_{5,2,7} + x_{6,2,7} + x_{7,2,7} + x_{8,2,7} \leq 1, x_{1,3,7} + x_{2,3,7} + x_{3,3,7} + x_{4,3,7} \\
& + x_{5,3,7} + x_{6,3,7} + x_{7,3,7} + x_{8,3,7} \leq 1, x_{1,4,7} + x_{2,4,7} + x_{3,4,7} + x_{4,4,7} + x_{5,4,7} \\
& + x_{6,4,7} + x_{7,4,7} + x_{8,4,7} \leq 1, x_{1,5,7} + x_{2,5,7} + x_{3,5,7} + x_{4,5,7} + x_{5,5,7} + x_{6,5,7} \\
& + x_{7,5,7} + x_{8,5,7} \leq 1, x_{1,6,7} + x_{2,6,7} + x_{3,6,7} + x_{4,6,7} + x_{5,6,7} + x_{6,6,7} + x_{7,6,7} \\
& + x_{8,6,7} \leq 1, x_{1,7,7} + x_{2,7,7} + x_{3,7,7} + x_{4,7,7} + x_{5,7,7} + x_{6,7,7} + x_{7,7,7} + x_{8,7,7}
\end{aligned}$$

$$\begin{aligned}
&\leq 1, x_{1,8,7} + x_{2,8,7} + x_{3,8,7} + x_{4,8,7} + x_{5,8,7} + x_{6,8,7} + x_{7,8,7} + x_{8,8,7} \leq 1, x_{1,1,8} \\
&+ x_{2,1,8} + x_{3,1,8} + x_{4,1,8} + x_{5,1,8} + x_{6,1,8} + x_{7,1,8} + x_{8,1,8} \leq 1, x_{1,2,8} + x_{2,2,8} \\
&+ x_{3,2,8} + x_{4,2,8} + x_{5,2,8} + x_{6,2,8} + x_{7,2,8} + x_{8,2,8} \leq 1, x_{1,3,8} + x_{2,3,8} + x_{3,3,8} \\
&+ x_{4,3,8} + x_{5,3,8} + x_{6,3,8} + x_{7,3,8} + x_{8,3,8} \leq 1, x_{1,4,8} + x_{2,4,8} + x_{3,4,8} + x_{4,4,8} \\
&+ x_{5,4,8} + x_{6,4,8} + x_{7,4,8} + x_{8,4,8} \leq 1, x_{1,5,8} + x_{2,5,8} + x_{3,5,8} + x_{4,5,8} + x_{5,5,8} \\
&+ x_{6,5,8} + x_{7,5,8} + x_{8,5,8} \leq 1, x_{1,6,8} + x_{2,6,8} + x_{3,6,8} + x_{4,6,8} + x_{5,6,8} + x_{6,6,8} \\
&+ x_{7,6,8} + x_{8,6,8} \leq 1, x_{1,7,8} + x_{2,7,8} + x_{3,7,8} + x_{4,7,8} + x_{5,7,8} + x_{6,7,8} + x_{7,7,8} \\
&+ x_{8,7,8} \leq 1, x_{1,8,8} + x_{2,8,8} + x_{3,8,8} + x_{4,8,8} + x_{5,8,8} + x_{6,8,8} + x_{7,8,8} + x_{8,8,8} \\
&\leq 1
\end{aligned}$$

> **VTConstraints := seq(seq(add(x[i,j,k], j=1..n) <= 1, i=1..n), k=1..m);**

$$\begin{aligned}
&VTConstraints := x_{1,1,1} + x_{1,2,1} + x_{1,3,1} + x_{1,4,1} + x_{1,5,1} + x_{1,6,1} + x_{1,7,1} + x_{1,8,1} \\
&\leq 1, x_{2,1,1} + x_{2,2,1} + x_{2,3,1} + x_{2,4,1} + x_{2,5,1} + x_{2,6,1} + x_{2,7,1} + x_{2,8,1} \leq 1, x_{3,1,1} \\
&+ x_{3,2,1} + x_{3,3,1} + x_{3,4,1} + x_{3,5,1} + x_{3,6,1} + x_{3,7,1} + x_{3,8,1} \leq 1, x_{4,1,1} + x_{4,2,1} \\
&+ x_{4,3,1} + x_{4,4,1} + x_{4,5,1} + x_{4,6,1} + x_{4,7,1} + x_{4,8,1} \leq 1, x_{5,1,1} + x_{5,2,1} + x_{5,3,1} \\
&+ x_{5,4,1} + x_{5,5,1} + x_{5,6,1} + x_{5,7,1} + x_{5,8,1} \leq 1, x_{6,1,1} + x_{6,2,1} + x_{6,3,1} + x_{6,4,1} \\
&+ x_{6,5,1} + x_{6,6,1} + x_{6,7,1} + x_{6,8,1} \leq 1, x_{7,1,1} + x_{7,2,1} + x_{7,3,1} + x_{7,4,1} + x_{7,5,1} \\
&+ x_{7,6,1} + x_{7,7,1} + x_{7,8,1} \leq 1, x_{8,1,1} + x_{8,2,1} + x_{8,3,1} + x_{8,4,1} + x_{8,5,1} + x_{8,6,1} \\
&+ x_{8,7,1} + x_{8,8,1} \leq 1, x_{1,1,2} + x_{1,2,2} + x_{1,3,2} + x_{1,4,2} + x_{1,5,2} + x_{1,6,2} + x_{1,7,2} \\
&+ x_{1,8,2} \leq 1, x_{2,1,2} + x_{2,2,2} + x_{2,3,2} + x_{2,4,2} + x_{2,5,2} + x_{2,6,2} + x_{2,7,2} + x_{2,8,2} \\
&\leq 1, x_{3,1,2} + x_{3,2,2} + x_{3,3,2} + x_{3,4,2} + x_{3,5,2} + x_{3,6,2} + x_{3,7,2} + x_{3,8,2} \leq 1, x_{4,1,2} \\
&+ x_{4,2,2} + x_{4,3,2} + x_{4,4,2} + x_{4,5,2} + x_{4,6,2} + x_{4,7,2} + x_{4,8,2} \leq 1, x_{5,1,2} + x_{5,2,2} \\
&+ x_{5,3,2} + x_{5,4,2} + x_{5,5,2} + x_{5,6,2} + x_{5,7,2} + x_{5,8,2} \leq 1, x_{6,1,2} + x_{6,2,2} + x_{6,3,2} \\
&+ x_{6,4,2} + x_{6,5,2} + x_{6,6,2} + x_{6,7,2} + x_{6,8,2} \leq 1, x_{7,1,2} + x_{7,2,2} + x_{7,3,2} + x_{7,4,2} \\
&+ x_{7,5,2} + x_{7,6,2} + x_{7,7,2} + x_{7,8,2} \leq 1, x_{8,1,2} + x_{8,2,2} + x_{8,3,2} + x_{8,4,2} + x_{8,5,2} \\
&+ x_{8,6,2} + x_{8,7,2} + x_{8,8,2} \leq 1, x_{1,1,3} + x_{1,2,3} + x_{1,3,3} + x_{1,4,3} + x_{1,5,3} + x_{1,6,3} \\
&+ x_{1,7,3} + x_{1,8,3} \leq 1, x_{2,1,3} + x_{2,2,3} + x_{2,3,3} + x_{2,4,3} + x_{2,5,3} + x_{2,6,3} + x_{2,7,3} \\
&+ x_{2,8,3} \leq 1, x_{3,1,3} + x_{3,2,3} + x_{3,3,3} + x_{3,4,3} + x_{3,5,3} + x_{3,6,3} + x_{3,7,3} + x_{3,8,3} \\
&\leq 1, x_{4,1,3} + x_{4,2,3} + x_{4,3,3} + x_{4,4,3} + x_{4,5,3} + x_{4,6,3} + x_{4,7,3} + x_{4,8,3} \leq 1, x_{5,1,3} \\
&+ x_{5,2,3} + x_{5,3,3} + x_{5,4,3} + x_{5,5,3} + x_{5,6,3} + x_{5,7,3} + x_{5,8,3} \leq 1, x_{6,1,3} + x_{6,2,3} \\
&+ x_{6,3,3} + x_{6,4,3} + x_{6,5,3} + x_{6,6,3} + x_{6,7,3} + x_{6,8,3} \leq 1, x_{7,1,3} + x_{7,2,3} + x_{7,3,3} \\
&+ x_{7,4,3} + x_{7,5,3} + x_{7,6,3} + x_{7,7,3} + x_{7,8,3} \leq 1, x_{8,1,3} + x_{8,2,3} + x_{8,3,3} + x_{8,4,3}
\end{aligned}$$

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$$\begin{aligned}
& + x_{8,5,3} + x_{8,6,3} + x_{8,7,3} + x_{8,8,3} \leq 1, x_{1,1,4} + x_{1,2,4} + x_{1,3,4} + x_{1,4,4} + x_{1,5,4} \\
& + x_{1,6,4} + x_{1,7,4} + x_{1,8,4} \leq 1, x_{2,1,4} + x_{2,2,4} + x_{2,3,4} + x_{2,4,4} + x_{2,5,4} + x_{2,6,4} \\
& + x_{2,7,4} + x_{2,8,4} \leq 1, x_{3,1,4} + x_{3,2,4} + x_{3,3,4} + x_{3,4,4} + x_{3,5,4} + x_{3,6,4} + x_{3,7,4} \\
& + x_{3,8,4} \leq 1, x_{4,1,4} + x_{4,2,4} + x_{4,3,4} + x_{4,4,4} + x_{4,5,4} + x_{4,6,4} + x_{4,7,4} + x_{4,8,4} \\
& \leq 1, x_{5,1,4} + x_{5,2,4} + x_{5,3,4} + x_{5,4,4} + x_{5,5,4} + x_{5,6,4} + x_{5,7,4} + x_{5,8,4} \leq 1, x_{6,1,4} \\
& + x_{6,2,4} + x_{6,3,4} + x_{6,4,4} + x_{6,5,4} + x_{6,6,4} + x_{6,7,4} + x_{6,8,4} \leq 1, x_{7,1,4} + x_{7,2,4} \\
& + x_{7,3,4} + x_{7,4,4} + x_{7,5,4} + x_{7,6,4} + x_{7,7,4} + x_{7,8,4} \leq 1, x_{8,1,4} + x_{8,2,4} + x_{8,3,4} \\
& + x_{8,4,4} + x_{8,5,4} + x_{8,6,4} + x_{8,7,4} + x_{8,8,4} \leq 1, x_{1,1,5} + x_{1,2,5} + x_{1,3,5} + x_{1,4,5} \\
& + x_{1,5,5} + x_{1,6,5} + x_{1,7,5} + x_{1,8,5} \leq 1, x_{2,1,5} + x_{2,2,5} + x_{2,3,5} + x_{2,4,5} + x_{2,5,5} \\
& + x_{2,6,5} + x_{2,7,5} + x_{2,8,5} \leq 1, x_{3,1,5} + x_{3,2,5} + x_{3,3,5} + x_{3,4,5} + x_{3,5,5} + x_{3,6,5} \\
& + x_{3,7,5} + x_{3,8,5} \leq 1, x_{4,1,5} + x_{4,2,5} + x_{4,3,5} + x_{4,4,5} + x_{4,5,5} + x_{4,6,5} + x_{4,7,5} \\
& + x_{4,8,5} \leq 1, x_{5,1,5} + x_{5,2,5} + x_{5,3,5} + x_{5,4,5} + x_{5,5,5} + x_{5,6,5} + x_{5,7,5} + x_{5,8,5} \\
& \leq 1, x_{6,1,5} + x_{6,2,5} + x_{6,3,5} + x_{6,4,5} + x_{6,5,5} + x_{6,6,5} + x_{6,7,5} + x_{6,8,5} \leq 1, x_{7,1,5} \\
& + x_{7,2,5} + x_{7,3,5} + x_{7,4,5} + x_{7,5,5} + x_{7,6,5} + x_{7,7,5} + x_{7,8,5} \leq 1, x_{8,1,5} + x_{8,2,5} \\
& + x_{8,3,5} + x_{8,4,5} + x_{8,5,5} + x_{8,6,5} + x_{8,7,5} + x_{8,8,5} \leq 1, x_{1,1,6} + x_{1,2,6} + x_{1,3,6} \\
& + x_{1,4,6} + x_{1,5,6} + x_{1,6,6} + x_{1,7,6} + x_{1,8,6} \leq 1, x_{2,1,6} + x_{2,2,6} + x_{2,3,6} + x_{2,4,6} \\
& + x_{2,5,6} + x_{2,6,6} + x_{2,7,6} + x_{2,8,6} \leq 1, x_{3,1,6} + x_{3,2,6} + x_{3,3,6} + x_{3,4,6} + x_{3,5,6} \\
& + x_{3,6,6} + x_{3,7,6} + x_{3,8,6} \leq 1, x_{4,1,6} + x_{4,2,6} + x_{4,3,6} + x_{4,4,6} + x_{4,5,6} + x_{4,6,6} \\
& + x_{4,7,6} + x_{4,8,6} \leq 1, x_{5,1,6} + x_{5,2,6} + x_{5,3,6} + x_{5,4,6} + x_{5,5,6} + x_{5,6,6} + x_{5,7,6} \\
& + x_{5,8,6} \leq 1, x_{6,1,6} + x_{6,2,6} + x_{6,3,6} + x_{6,4,6} + x_{6,5,6} + x_{6,6,6} + x_{6,7,6} + x_{6,8,6} \\
& \leq 1, x_{7,1,6} + x_{7,2,6} + x_{7,3,6} + x_{7,4,6} + x_{7,5,6} + x_{7,6,6} + x_{7,7,6} + x_{7,8,6} \leq 1, x_{8,1,6} \\
& + x_{8,2,6} + x_{8,3,6} + x_{8,4,6} + x_{8,5,6} + x_{8,6,6} + x_{8,7,6} + x_{8,8,6} \leq 1, x_{1,1,7} + x_{1,2,7} \\
& + x_{1,3,7} + x_{1,4,7} + x_{1,5,7} + x_{1,6,7} + x_{1,7,7} + x_{1,8,7} \leq 1, x_{2,1,7} + x_{2,2,7} + x_{2,3,7} \\
& + x_{2,4,7} + x_{2,5,7} + x_{2,6,7} + x_{2,7,7} + x_{2,8,7} \leq 1, x_{3,1,7} + x_{3,2,7} + x_{3,3,7} + x_{3,4,7} \\
& + x_{3,5,7} + x_{3,6,7} + x_{3,7,7} + x_{3,8,7} \leq 1, x_{4,1,7} + x_{4,2,7} + x_{4,3,7} + x_{4,4,7} + x_{4,5,7} \\
& + x_{4,6,7} + x_{4,7,7} + x_{4,8,7} \leq 1, x_{5,1,7} + x_{5,2,7} + x_{5,3,7} + x_{5,4,7} + x_{5,5,7} + x_{5,6,7} \\
& + x_{5,7,7} + x_{5,8,7} \leq 1, x_{6,1,7} + x_{6,2,7} + x_{6,3,7} + x_{6,4,7} + x_{6,5,7} + x_{6,6,7} + x_{6,7,7} \\
& + x_{6,8,7} \leq 1, x_{7,1,7} + x_{7,2,7} + x_{7,3,7} + x_{7,4,7} + x_{7,5,7} + x_{7,6,7} + x_{7,7,7} + x_{7,8,7} \\
& \leq 1, x_{8,1,7} + x_{8,2,7} + x_{8,3,7} + x_{8,4,7} + x_{8,5,7} + x_{8,6,7} + x_{8,7,7} + x_{8,8,7} \leq 1, x_{1,1,8} \\
& + x_{1,2,8} + x_{1,3,8} + x_{1,4,8} + x_{1,5,8} + x_{1,6,8} + x_{1,7,8} + x_{1,8,8} \leq 1, x_{2,1,8} + x_{2,2,8} \\
& + x_{2,3,8} + x_{2,4,8} + x_{2,5,8} + x_{2,6,8} + x_{2,7,8} + x_{2,8,8} \leq 1, x_{3,1,8} + x_{3,2,8} + x_{3,3,8}
\end{aligned}$$

$$\begin{aligned}
& + x_{3,4,8} + x_{3,5,8} + x_{3,6,8} + x_{3,7,8} + x_{3,8,8} \leq 1, x_{4,1,8} + x_{4,2,8} + x_{4,3,8} + x_{4,4,8} \\
& + x_{4,5,8} + x_{4,6,8} + x_{4,7,8} + x_{4,8,8} \leq 1, x_{5,1,8} + x_{5,2,8} + x_{5,3,8} + x_{5,4,8} + x_{5,5,8} \\
& + x_{5,6,8} + x_{5,7,8} + x_{5,8,8} \leq 1, x_{6,1,8} + x_{6,2,8} + x_{6,3,8} + x_{6,4,8} + x_{6,5,8} + x_{6,6,8} \\
& + x_{6,7,8} + x_{6,8,8} \leq 1, x_{7,1,8} + x_{7,2,8} + x_{7,3,8} + x_{7,4,8} + x_{7,5,8} + x_{7,6,8} + x_{7,7,8} \\
& + x_{7,8,8} \leq 1, x_{8,1,8} + x_{8,2,8} + x_{8,3,8} + x_{8,4,8} + x_{8,5,8} + x_{8,6,8} + x_{8,7,8} + x_{8,8,8} \\
& \leq 1
\end{aligned}$$

> **TConstraints2:=seq(seq(x[i,i,k]=0,i=1..n),k=1..m);**

$$\begin{aligned}
TConstraints2 := & x_{1,1,1} = 0, x_{2,2,1} = 0, x_{3,3,1} = 0, x_{4,4,1} = 0, x_{5,5,1} = 0, x_{6,6,1} = 0, x_{7,7,1} \\
& = 0, x_{8,8,1} = 0, x_{1,1,2} = 0, x_{2,2,2} = 0, x_{3,3,2} = 0, x_{4,4,2} = 0, x_{5,5,2} = 0, x_{6,6,2} = 0, x_{7,7,2} \\
& = 0, x_{8,8,2} = 0, x_{1,1,3} = 0, x_{2,2,3} = 0, x_{3,3,3} = 0, x_{4,4,3} = 0, x_{5,5,3} = 0, x_{6,6,3} = 0, x_{7,7,3} \\
& = 0, x_{8,8,3} = 0, x_{1,1,4} = 0, x_{2,2,4} = 0, x_{3,3,4} = 0, x_{4,4,4} = 0, x_{5,5,4} = 0, x_{6,6,4} = 0, x_{7,7,4} \\
& = 0, x_{8,8,4} = 0, x_{1,1,5} = 0, x_{2,2,5} = 0, x_{3,3,5} = 0, x_{4,4,5} = 0, x_{5,5,5} = 0, x_{6,6,5} = 0, x_{7,7,5} \\
& = 0, x_{8,8,5} = 0, x_{1,1,6} = 0, x_{2,2,6} = 0, x_{3,3,6} = 0, x_{4,4,6} = 0, x_{5,5,6} = 0, x_{6,6,6} = 0, x_{7,7,6} \\
& = 0, x_{8,8,6} = 0, x_{1,1,7} = 0, x_{2,2,7} = 0, x_{3,3,7} = 0, x_{4,4,7} = 0, x_{5,5,7} = 0, x_{6,6,7} = 0, x_{7,7,7} \\
& = 0, x_{8,8,7} = 0, x_{1,1,8} = 0, x_{2,2,8} = 0, x_{3,3,8} = 0, x_{4,4,8} = 0, x_{5,5,8} = 0, x_{6,6,8} = 0, x_{7,7,8} \\
& = 0, x_{8,8,8} = 0
\end{aligned}$$

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> **Objective:=add(add(add(x[i,j,k],j=1..n),i=1..n),k=1..m);**

$$\begin{aligned}
Objective := & x_{2,1,4} + x_{1,2,4} + x_{3,1,4} + x_{1,3,4} + x_{4,1,4} + x_{1,4,4} + x_{2,2,4} + x_{3,2,4} + x_{2,3,4} \\
& + x_{4,2,4} + x_{2,4,4} + x_{3,3,4} + x_{4,3,4} + x_{3,4,4} + x_{4,4,4} + x_{3,1,1} + x_{1,3,1} + x_{4,1,1} \\
& + x_{1,4,1} + x_{2,2,1} + x_{3,2,1} + x_{2,3,1} + x_{4,2,1} + x_{2,4,1} + x_{3,3,1} + x_{4,3,1} + x_{3,4,1} \\
& + x_{4,4,1} + x_{1,1,2} + x_{2,1,2} + x_{1,2,2} + x_{3,1,2} + x_{1,3,2} + x_{4,1,2} + x_{1,4,2} + x_{2,2,2} \\
& + x_{3,2,2} + x_{2,3,2} + x_{4,2,2} + x_{2,4,2} + x_{3,3,2} + x_{4,3,2} + x_{3,4,2} + x_{4,4,2} + x_{1,1,3} \\
& + x_{2,1,3} + x_{1,2,3} + x_{3,1,3} + x_{1,3,3} + x_{4,1,3} + x_{1,4,3} + x_{2,2,3} + x_{3,2,3} + x_{2,3,3} \\
& + x_{4,2,3} + x_{2,4,3} + x_{3,3,3} + x_{4,3,3} + x_{3,4,3} + x_{4,4,3} + x_{1,1,4} + x_{1,1,1} + x_{2,1,1} \\
& + x_{1,2,1} + x_{4,5,8} + x_{5,5,8} + x_{6,5,8} + x_{7,5,8} + x_{8,5,8} + x_{1,6,8} + x_{2,6,8} + x_{3,6,8} \\
& + x_{4,6,8} + x_{5,6,8} + x_{6,6,8} + x_{7,6,8} + x_{8,6,8} + x_{1,7,8} + x_{2,7,8} + x_{3,7,8} + x_{4,7,8} \\
& + x_{5,7,8} + x_{6,7,8} + x_{7,7,8} + x_{8,7,8} + x_{1,8,8} + x_{2,8,8} + x_{3,8,8} + x_{4,8,8} + x_{5,8,8} \\
& + x_{6,8,8} + x_{7,8,8} + x_{8,8,8} + x_{1,4,7} + x_{2,4,7} + x_{3,4,7} + x_{4,4,7} + x_{5,4,7} + x_{6,4,7} \\
& + x_{7,4,7} + x_{8,4,7} + x_{1,5,7} + x_{2,5,7} + x_{3,5,7} + x_{4,5,7} + x_{5,5,7} + x_{6,5,7} + x_{7,5,7} \\
& + x_{8,5,7} + x_{1,6,7} + x_{2,6,7} + x_{3,6,7} + x_{4,6,7} + x_{5,6,7} + x_{6,6,7} + x_{7,6,7} + x_{8,6,7}
\end{aligned}$$

(10)

$$\begin{aligned}
& + x_{1,7,7} + x_{2,7,7} + x_{3,7,7} + x_{4,7,7} + x_{5,7,7} + x_{6,7,7} + x_{7,7,7} + x_{8,7,7} + x_{1,8,7} \\
& + x_{2,8,7} + x_{3,8,7} + x_{4,8,7} + x_{5,8,7} + x_{6,8,7} + x_{7,8,7} + x_{8,8,7} + x_{1,1,8} + x_{2,1,8} \\
& + x_{3,1,8} + x_{4,1,8} + x_{5,1,8} + x_{6,1,8} + x_{7,1,8} + x_{8,1,8} + x_{1,2,8} + x_{2,2,8} + x_{3,2,8} \\
& + x_{4,2,8} + x_{5,2,8} + x_{6,2,8} + x_{7,2,8} + x_{8,2,8} + x_{1,3,8} + x_{2,3,8} + x_{3,3,8} + x_{4,3,8} \\
& + x_{5,3,8} + x_{6,3,8} + x_{7,3,8} + x_{8,3,8} + x_{1,4,8} + x_{2,4,8} + x_{3,4,8} + x_{4,4,8} + x_{5,4,8} \\
& + x_{6,4,8} + x_{7,4,8} + x_{8,4,8} + x_{1,5,8} + x_{2,5,8} + x_{3,5,8} + x_{7,2,6} + x_{8,2,6} + x_{1,3,6} \\
& + x_{2,3,6} + x_{3,3,6} + x_{4,3,6} + x_{5,3,6} + x_{6,3,6} + x_{7,3,6} + x_{8,3,6} + x_{1,4,6} + x_{2,4,6} \\
& + x_{3,4,6} + x_{4,4,6} + x_{5,4,6} + x_{6,4,6} + x_{7,4,6} + x_{8,4,6} + x_{1,5,6} + x_{2,5,6} + x_{3,5,6} \\
& + x_{4,5,6} + x_{5,5,6} + x_{6,5,6} + x_{7,5,6} + x_{8,5,6} + x_{1,6,6} + x_{2,6,6} + x_{3,6,6} + x_{4,6,6} \\
& + x_{5,6,6} + x_{6,6,6} + x_{7,6,6} + x_{8,6,6} + x_{1,7,6} + x_{2,7,6} + x_{3,7,6} + x_{4,7,6} + x_{5,7,6} \\
& + x_{6,7,6} + x_{7,7,6} + x_{8,7,6} + x_{1,8,6} + x_{2,8,6} + x_{3,8,6} + x_{4,8,6} + x_{5,8,6} + x_{6,8,6} \\
& + x_{7,8,6} + x_{8,8,6} + x_{1,1,7} + x_{2,1,7} + x_{3,1,7} + x_{4,1,7} + x_{5,1,7} + x_{6,1,7} + x_{7,1,7} \\
& + x_{8,1,7} + x_{1,2,7} + x_{2,2,7} + x_{3,2,7} + x_{4,2,7} + x_{5,2,7} + x_{6,2,7} + x_{7,2,7} + x_{8,2,7} \\
& + x_{1,3,7} + x_{2,3,7} + x_{3,3,7} + x_{4,3,7} + x_{5,3,7} + x_{6,3,7} + x_{7,3,7} + x_{8,3,7} + x_{4,1,5} \\
& + x_{5,1,5} + x_{6,1,5} + x_{7,1,5} + x_{8,1,5} + x_{1,2,5} + x_{2,2,5} + x_{3,2,5} + x_{4,2,5} + x_{5,2,5} \\
& + x_{6,2,5} + x_{7,2,5} + x_{8,2,5} + x_{1,3,5} + x_{2,3,5} + x_{3,3,5} + x_{4,3,5} + x_{5,3,5} + x_{6,3,5} \\
& + x_{7,3,5} + x_{8,3,5} + x_{1,4,5} + x_{2,4,5} + x_{3,4,5} + x_{4,4,5} + x_{5,4,5} + x_{6,4,5} + x_{7,4,5} \\
& + x_{8,4,5} + x_{1,5,5} + x_{2,5,5} + x_{3,5,5} + x_{4,5,5} + x_{5,5,5} + x_{6,5,5} + x_{7,5,5} + x_{8,5,5} \\
& + x_{1,6,5} + x_{2,6,5} + x_{3,6,5} + x_{4,6,5} + x_{5,6,5} + x_{6,6,5} + x_{7,6,5} + x_{8,6,5} + x_{1,7,5} \\
& + x_{2,7,5} + x_{3,7,5} + x_{4,7,5} + x_{5,7,5} + x_{6,7,5} + x_{7,7,5} + x_{8,7,5} + x_{1,8,5} + x_{2,8,5} \\
& + x_{3,8,5} + x_{4,8,5} + x_{5,8,5} + x_{6,8,5} + x_{7,8,5} + x_{8,8,5} + x_{1,1,6} + x_{2,1,6} + x_{3,1,6} \\
& + x_{4,1,6} + x_{5,1,6} + x_{6,1,6} + x_{7,1,6} + x_{8,1,6} + x_{1,2,6} + x_{2,2,6} + x_{3,2,6} + x_{4,2,6} \\
& + x_{5,2,6} + x_{6,2,6} + x_{2,8,3} + x_{3,8,3} + x_{4,8,3} + x_{5,8,3} + x_{6,8,3} + x_{7,8,3} + x_{8,8,3} \\
& + x_{5,1,4} + x_{6,1,4} + x_{7,1,4} + x_{8,1,4} + x_{5,2,4} + x_{6,2,4} + x_{7,2,4} + x_{8,2,4} + x_{5,3,4} \\
& + x_{6,3,4} + x_{7,3,4} + x_{8,3,4} + x_{5,4,4} + x_{6,4,4} + x_{7,4,4} + x_{8,4,4} + x_{1,5,4} + x_{2,5,4} \\
& + x_{3,5,4} + x_{4,5,4} + x_{5,5,4} + x_{6,5,4} + x_{7,5,4} + x_{8,5,4} + x_{1,6,4} + x_{2,6,4} + x_{3,6,4} \\
& + x_{4,6,4} + x_{5,6,4} + x_{6,6,4} + x_{7,6,4} + x_{8,6,4} + x_{1,7,4} + x_{2,7,4} + x_{3,7,4} + x_{4,7,4} \\
& + x_{5,7,4} + x_{6,7,4} + x_{7,7,4} + x_{8,7,4} + x_{1,8,4} + x_{2,8,4} + x_{3,8,4} + x_{4,8,4} + x_{5,8,4} \\
& + x_{6,8,4} + x_{7,8,4} + x_{8,8,4} + x_{1,1,5} + x_{2,1,5} + x_{3,1,5} + x_{8,6,2} + x_{1,7,2} + x_{2,7,2} \\
& + x_{3,7,2} + x_{4,7,2} + x_{5,7,2} + x_{6,7,2} + x_{7,7,2} + x_{8,7,2} + x_{1,8,2} + x_{2,8,2} + x_{3,8,2} \\
& + x_{4,8,2} + x_{5,8,2} + x_{6,8,2} + x_{7,8,2} + x_{8,8,2} + x_{5,1,3} + x_{6,1,3} + x_{7,1,3} + x_{8,1,3}
\end{aligned}$$

$$\begin{aligned}
& + x_{5,2,3} + x_{6,2,3} + x_{7,2,3} + x_{8,2,3} + x_{5,3,3} + x_{6,3,3} + x_{7,3,3} + x_{8,3,3} + x_{5,4,3} \\
& + x_{6,4,3} + x_{7,4,3} + x_{8,4,3} + x_{1,5,3} + x_{2,5,3} + x_{3,5,3} + x_{4,5,3} + x_{5,5,3} + x_{6,5,3} \\
& + x_{7,5,3} + x_{8,5,3} + x_{1,6,3} + x_{2,6,3} + x_{3,6,3} + x_{4,6,3} + x_{5,6,3} + x_{6,6,3} + x_{7,6,3} \\
& + x_{8,6,3} + x_{1,7,3} + x_{2,7,3} + x_{3,7,3} + x_{4,7,3} + x_{5,7,3} + x_{6,7,3} + x_{7,7,3} + x_{8,7,3} \\
& + x_{1,8,3} + x_{5,5,1} + x_{6,5,1} + x_{7,5,1} + x_{8,5,1} + x_{1,6,1} + x_{2,6,1} + x_{3,6,1} + x_{4,6,1} \\
& + x_{5,6,1} + x_{6,6,1} + x_{7,6,1} + x_{8,6,1} + x_{1,7,1} + x_{2,7,1} + x_{3,7,1} + x_{4,7,1} + x_{5,7,1} \\
& + x_{6,7,1} + x_{7,7,1} + x_{8,7,1} + x_{1,8,1} + x_{2,8,1} + x_{3,8,1} + x_{4,8,1} + x_{5,8,1} + x_{6,8,1} \\
& + x_{7,8,1} + x_{8,8,1} + x_{5,1,2} + x_{6,1,2} + x_{7,1,2} + x_{8,1,2} + x_{5,2,2} + x_{6,2,2} + x_{7,2,2} \\
& + x_{8,2,2} + x_{5,3,2} + x_{6,3,2} + x_{7,3,2} + x_{8,3,2} + x_{5,4,2} + x_{6,4,2} + x_{7,4,2} + x_{8,4,2} \\
& + x_{1,5,2} + x_{2,5,2} + x_{3,5,2} + x_{4,5,2} + x_{5,5,2} + x_{6,5,2} + x_{7,5,2} + x_{8,5,2} + x_{1,6,2} \\
& + x_{2,6,2} + x_{3,6,2} + x_{4,6,2} + x_{5,6,2} + x_{6,6,2} + x_{7,6,2} + x_{5,1,1} + x_{6,1,1} + x_{7,1,1} \\
& + x_{8,1,1} + x_{5,2,1} + x_{6,2,1} + x_{7,2,1} + x_{8,2,1} + x_{5,3,1} + x_{6,3,1} + x_{7,3,1} + x_{8,3,1} \\
& + x_{5,4,1} + x_{6,4,1} + x_{7,4,1} + x_{8,4,1} + x_{1,5,1} + x_{2,5,1} + x_{3,5,1} + x_{4,5,1}
\end{aligned}$$

> Sol:=LPSolve(Objective, [TConstraints2,More,VTConstraints, HTConstraints,ByeWeeks1,ByeWeeks2],assume={'nonnegative','integer'}, maximize=true);

$$\begin{aligned}
\text{Sol} := & [56, [x_{1,1,1} = 0, x_{1,1,2} = 0, x_{1,1,3} = 0, x_{1,1,4} = 0, x_{1,1,5} = 0, x_{1,1,6} = 0, x_{1,1,7} = 0, \\
& x_{1,1,8} = 0, x_{1,2,1} = 0, x_{1,2,2} = 0, x_{1,2,3} = 0, x_{1,2,4} = 1, x_{1,2,5} = 1, x_{1,2,6} = 1, x_{1,2,7} = 0, \\
& x_{1,2,8} = 1, x_{1,3,1} = 0, x_{1,3,2} = 0, x_{1,3,3} = 1, x_{1,3,4} = 0, x_{1,3,5} = 0, x_{1,3,6} = 0, x_{1,3,7} = 1, \\
& x_{1,3,8} = 0, x_{1,4,1} = 0, x_{1,4,2} = 0, x_{1,4,3} = 0, x_{1,4,4} = 0, x_{1,4,5} = 0, x_{1,4,6} = 0, x_{1,4,7} = 0, \\
& x_{1,4,8} = 0, x_{1,5,1} = 0, x_{1,5,2} = 1, x_{1,5,3} = 0, x_{1,5,4} = 0, x_{1,5,5} = 0, x_{1,5,6} = 0, x_{1,5,7} = 0, \\
& x_{1,5,8} = 0, x_{1,6,1} = 0, x_{1,6,2} = 0, x_{1,6,3} = 0, x_{1,6,4} = 0, x_{1,6,5} = 0, x_{1,6,6} = 0, x_{1,6,7} = 0, \\
& x_{1,6,8} = 0, x_{1,7,1} = 0, x_{1,7,2} = 0, x_{1,7,3} = 0, x_{1,7,4} = 0, x_{1,7,5} = 0, x_{1,7,6} = 0, x_{1,7,7} = 0, \\
& x_{1,7,8} = 0, x_{1,8,1} = 0, x_{1,8,2} = 0, x_{1,8,3} = 0, x_{1,8,4} = 0, x_{1,8,5} = 0, x_{1,8,6} = 0, x_{1,8,7} = 0, \\
& x_{1,8,8} = 0, x_{2,1,1} = 0, x_{2,1,2} = 0, x_{2,1,3} = 0, x_{2,1,4} = 1, x_{2,1,5} = 1, x_{2,1,6} = 1, x_{2,1,7} = 0, \\
& x_{2,1,8} = 1, x_{2,2,1} = 0, x_{2,2,2} = 0, x_{2,2,3} = 0, x_{2,2,4} = 0, x_{2,2,5} = 0, x_{2,2,6} = 0, x_{2,2,7} = 0, \\
& x_{2,2,8} = 0, x_{2,3,1} = 0, x_{2,3,2} = 1, x_{2,3,3} = 0, x_{2,3,4} = 0, x_{2,3,5} = 0, x_{2,3,6} = 0, x_{2,3,7} = 0, \\
& x_{2,3,8} = 0, x_{2,4,1} = 0, x_{2,4,2} = 0, x_{2,4,3} = 1, x_{2,4,4} = 0, x_{2,4,5} = 0, x_{2,4,6} = 0, x_{2,4,7} = 0, \\
& x_{2,4,8} = 0, x_{2,5,1} = 0, x_{2,5,2} = 0, x_{2,5,3} = 0, x_{2,5,4} = 0, x_{2,5,5} = 0, x_{2,5,6} = 0, x_{2,5,7} = 1, \\
& x_{2,5,8} = 0, x_{2,6,1} = 0, x_{2,6,2} = 0, x_{2,6,3} = 0, x_{2,6,4} = 0, x_{2,6,5} = 0, x_{2,6,6} = 0, x_{2,6,7} = 0, \\
& x_{2,6,8} = 0, x_{2,7,1} = 0, x_{2,7,2} = 0, x_{2,7,3} = 0, x_{2,7,4} = 0, x_{2,7,5} = 0, x_{2,7,6} = 0, x_{2,7,7} = 0, \\
& x_{2,7,8} = 0, x_{2,8,1} = 0, x_{2,8,2} = 0, x_{2,8,3} = 0, x_{2,8,4} = 0, x_{2,8,5} = 0, x_{2,8,6} = 0, x_{2,8,7} = 0,
\end{aligned} \tag{11}$$

$$\begin{aligned}
&x_{2,8,8} = 0, x_{3,1,1} = 0, x_{3,1,2} = 0, x_{3,1,3} = 1, x_{3,1,4} = 0, x_{3,1,5} = 0, x_{3,1,6} = 0, x_{3,1,7} = 1, \\
&x_{3,1,8} = 0, x_{3,2,1} = 0, x_{3,2,2} = 1, x_{3,2,3} = 0, x_{3,2,4} = 0, x_{3,2,5} = 0, x_{3,2,6} = 0, x_{3,2,7} = 0, \\
&x_{3,2,8} = 0, x_{3,3,1} = 0, x_{3,3,2} = 0, x_{3,3,3} = 0, x_{3,3,4} = 0, x_{3,3,5} = 0, x_{3,3,6} = 0, x_{3,3,7} = 0, \\
&x_{3,3,8} = 0, x_{3,4,1} = 0, x_{3,4,2} = 0, x_{3,4,3} = 0, x_{3,4,4} = 1, x_{3,4,5} = 0, x_{3,4,6} = 1, x_{3,4,7} = 0, \\
&x_{3,4,8} = 0, x_{3,5,1} = 1, x_{3,5,2} = 0, x_{3,5,3} = 0, x_{3,5,4} = 0, x_{3,5,5} = 0, x_{3,5,6} = 0, x_{3,5,7} = 0, \\
&x_{3,5,8} = 0, x_{3,6,1} = 0, x_{3,6,2} = 0, x_{3,6,3} = 0, x_{3,6,4} = 0, x_{3,6,5} = 1, x_{3,6,6} = 0, x_{3,6,7} = 0, \\
&x_{3,6,8} = 0, x_{3,7,1} = 0, x_{3,7,2} = 0, x_{3,7,3} = 0, x_{3,7,4} = 0, x_{3,7,5} = 0, x_{3,7,6} = 0, x_{3,7,7} = 0, \\
&x_{3,7,8} = 0, x_{3,8,1} = 0, x_{3,8,2} = 0, x_{3,8,3} = 0, x_{3,8,4} = 0, x_{3,8,5} = 0, x_{3,8,6} = 0, x_{3,8,7} = 0, \\
&x_{3,8,8} = 0, x_{4,1,1} = 0, x_{4,1,2} = 0, x_{4,1,3} = 0, x_{4,1,4} = 0, x_{4,1,5} = 0, x_{4,1,6} = 0, x_{4,1,7} = 0, \\
&x_{4,1,8} = 0, x_{4,2,1} = 0, x_{4,2,2} = 0, x_{4,2,3} = 1, x_{4,2,4} = 0, x_{4,2,5} = 0, x_{4,2,6} = 0, x_{4,2,7} = 0, \\
&x_{4,2,8} = 0, x_{4,3,1} = 0, x_{4,3,2} = 0, x_{4,3,3} = 0, x_{4,3,4} = 1, x_{4,3,5} = 0, x_{4,3,6} = 1, x_{4,3,7} = 0, \\
&x_{4,3,8} = 0, x_{4,4,1} = 0, x_{4,4,2} = 0, x_{4,4,3} = 0, x_{4,4,4} = 0, x_{4,4,5} = 0, x_{4,4,6} = 0, x_{4,4,7} = 0, \\
&x_{4,4,8} = 0, x_{4,5,1} = 0, x_{4,5,2} = 0, x_{4,5,3} = 0, x_{4,5,4} = 0, x_{4,5,5} = 1, x_{4,5,6} = 0, x_{4,5,7} = 0, \\
&x_{4,5,8} = 0, x_{4,6,1} = 1, x_{4,6,2} = 0, x_{4,6,3} = 0, x_{4,6,4} = 0, x_{4,6,5} = 0, x_{4,6,6} = 0, x_{4,6,7} = 0, \\
&x_{4,6,8} = 0, x_{4,7,1} = 0, x_{4,7,2} = 0, x_{4,7,3} = 0, x_{4,7,4} = 0, x_{4,7,5} = 0, x_{4,7,6} = 0, x_{4,7,7} = 1, \\
&x_{4,7,8} = 0, x_{4,8,1} = 0, x_{4,8,2} = 1, x_{4,8,3} = 0, x_{4,8,4} = 0, x_{4,8,5} = 0, x_{4,8,6} = 0, x_{4,8,7} = 0, \\
&x_{4,8,8} = 0, x_{5,1,1} = 0, x_{5,1,2} = 1, x_{5,1,3} = 0, x_{5,1,4} = 0, x_{5,1,5} = 0, x_{5,1,6} = 0, x_{5,1,7} = 0, \\
&x_{5,1,8} = 0, x_{5,2,1} = 0, x_{5,2,2} = 0, x_{5,2,3} = 0, x_{5,2,4} = 0, x_{5,2,5} = 0, x_{5,2,6} = 0, x_{5,2,7} = 1, \\
&x_{5,2,8} = 0, x_{5,3,1} = 1, x_{5,3,2} = 0, x_{5,3,3} = 0, x_{5,3,4} = 0, x_{5,3,5} = 0, x_{5,3,6} = 0, x_{5,3,7} = 0, \\
&x_{5,3,8} = 0, x_{5,4,1} = 0, x_{5,4,2} = 0, x_{5,4,3} = 0, x_{5,4,4} = 0, x_{5,4,5} = 1, x_{5,4,6} = 0, x_{5,4,7} = 0, \\
&x_{5,4,8} = 0, x_{5,5,1} = 0, x_{5,5,2} = 0, x_{5,5,3} = 0, x_{5,5,4} = 0, x_{5,5,5} = 0, x_{5,5,6} = 0, x_{5,5,7} = 0, \\
&x_{5,5,8} = 0, x_{5,6,1} = 0, x_{5,6,2} = 0, x_{5,6,3} = 0, x_{5,6,4} = 1, x_{5,6,5} = 0, x_{5,6,6} = 0, x_{5,6,7} = 0, \\
&x_{5,6,8} = 1, x_{5,7,1} = 0, x_{5,7,2} = 0, x_{5,7,3} = 0, x_{5,7,4} = 0, x_{5,7,5} = 0, x_{5,7,6} = 1, x_{5,7,7} = 0, \\
&x_{5,7,8} = 0, x_{5,8,1} = 0, x_{5,8,2} = 0, x_{5,8,3} = 0, x_{5,8,4} = 0, x_{5,8,5} = 0, x_{5,8,6} = 0, x_{5,8,7} = 0, \\
&x_{5,8,8} = 0, x_{6,1,1} = 0, x_{6,1,2} = 0, x_{6,1,3} = 0, x_{6,1,4} = 0, x_{6,1,5} = 0, x_{6,1,6} = 0, x_{6,1,7} = 0, \\
&x_{6,1,8} = 0, x_{6,2,1} = 0, x_{6,2,2} = 0, x_{6,2,3} = 0, x_{6,2,4} = 0, x_{6,2,5} = 0, x_{6,2,6} = 0, x_{6,2,7} = 0, \\
&x_{6,2,8} = 0, x_{6,3,1} = 0, x_{6,3,2} = 0, x_{6,3,3} = 0, x_{6,3,4} = 0, x_{6,3,5} = 1, x_{6,3,6} = 0, x_{6,3,7} = 0, \\
&x_{6,3,8} = 0, x_{6,4,1} = 1, x_{6,4,2} = 0, x_{6,4,3} = 0, x_{6,4,4} = 0, x_{6,4,5} = 0, x_{6,4,6} = 0, x_{6,4,7} = 0, \\
&x_{6,4,8} = 0, x_{6,5,1} = 0, x_{6,5,2} = 0, x_{6,5,3} = 0, x_{6,5,4} = 1, x_{6,5,5} = 0, x_{6,5,6} = 0, x_{6,5,7} = 0, \\
&x_{6,5,8} = 1, x_{6,6,1} = 0, x_{6,6,2} = 0, x_{6,6,3} = 0, x_{6,6,4} = 0, x_{6,6,5} = 0, x_{6,6,6} = 0, x_{6,6,7} = 0, \\
&x_{6,6,8} = 0, x_{6,7,1} = 0, x_{6,7,2} = 1, x_{6,7,3} = 0, x_{6,7,4} = 0, x_{6,7,5} = 0, x_{6,7,6} = 0, x_{6,7,7} = 0,
\end{aligned}$$

$x_{6,7,8} = 0, x_{6,8,1} = 0, x_{6,8,2} = 0, x_{6,8,3} = 0, x_{6,8,4} = 0, x_{6,8,5} = 0, x_{6,8,6} = 1, x_{6,8,7} = 1,$
 $x_{6,8,8} = 0, x_{7,1,1} = 0, x_{7,1,2} = 0, x_{7,1,3} = 0, x_{7,1,4} = 0, x_{7,1,5} = 0, x_{7,1,6} = 0, x_{7,1,7} = 0,$
 $x_{7,1,8} = 0, x_{7,2,1} = 0, x_{7,2,2} = 0, x_{7,2,3} = 0, x_{7,2,4} = 0, x_{7,2,5} = 0, x_{7,2,6} = 0, x_{7,2,7} = 0,$
 $x_{7,2,8} = 0, x_{7,3,1} = 0, x_{7,3,2} = 0, x_{7,3,3} = 0, x_{7,3,4} = 0, x_{7,3,5} = 0, x_{7,3,6} = 0, x_{7,3,7} = 0,$
 $x_{7,3,8} = 0, x_{7,4,1} = 0, x_{7,4,2} = 0, x_{7,4,3} = 0, x_{7,4,4} = 0, x_{7,4,5} = 0, x_{7,4,6} = 0, x_{7,4,7} = 1,$
 $x_{7,4,8} = 0, x_{7,5,1} = 0, x_{7,5,2} = 0, x_{7,5,3} = 0, x_{7,5,4} = 0, x_{7,5,5} = 0, x_{7,5,6} = 1, x_{7,5,7} = 0,$
 $x_{7,5,8} = 0, x_{7,6,1} = 0, x_{7,6,2} = 1, x_{7,6,3} = 0, x_{7,6,4} = 0, x_{7,6,5} = 0, x_{7,6,6} = 0, x_{7,6,7} = 0,$
 $x_{7,6,8} = 0, x_{7,7,1} = 0, x_{7,7,2} = 0, x_{7,7,3} = 0, x_{7,7,4} = 0, x_{7,7,5} = 0, x_{7,7,6} = 0, x_{7,7,7} = 0,$
 $x_{7,7,8} = 0, x_{7,8,1} = 1, x_{7,8,2} = 0, x_{7,8,3} = 1, x_{7,8,4} = 0, x_{7,8,5} = 1, x_{7,8,6} = 0, x_{7,8,7} = 0,$
 $x_{7,8,8} = 1, x_{8,1,1} = 0, x_{8,1,2} = 0, x_{8,1,3} = 0, x_{8,1,4} = 0, x_{8,1,5} = 0, x_{8,1,6} = 0, x_{8,1,7} = 0,$
 $x_{8,1,8} = 0, x_{8,2,1} = 0, x_{8,2,2} = 0, x_{8,2,3} = 0, x_{8,2,4} = 0, x_{8,2,5} = 0, x_{8,2,6} = 0, x_{8,2,7} = 0,$
 $x_{8,2,8} = 0, x_{8,3,1} = 0, x_{8,3,2} = 0, x_{8,3,3} = 0, x_{8,3,4} = 0, x_{8,3,5} = 0, x_{8,3,6} = 0, x_{8,3,7} = 0,$
 $x_{8,3,8} = 0, x_{8,4,1} = 0, x_{8,4,2} = 1, x_{8,4,3} = 0, x_{8,4,4} = 0, x_{8,4,5} = 0, x_{8,4,6} = 0, x_{8,4,7} = 0,$
 $x_{8,4,8} = 0, x_{8,5,1} = 0, x_{8,5,2} = 0, x_{8,5,3} = 0, x_{8,5,4} = 0, x_{8,5,5} = 0, x_{8,5,6} = 0, x_{8,5,7} = 0,$
 $x_{8,5,8} = 0, x_{8,6,1} = 0, x_{8,6,2} = 0, x_{8,6,3} = 0, x_{8,6,4} = 0, x_{8,6,5} = 0, x_{8,6,6} = 1, x_{8,6,7} = 1,$
 $x_{8,6,8} = 0, x_{8,7,1} = 1, x_{8,7,2} = 0, x_{8,7,3} = 1, x_{8,7,4} = 0, x_{8,7,5} = 1, x_{8,7,6} = 0, x_{8,7,7} = 0,$
 $x_{8,7,8} = 1, x_{8,8,1} = 0, x_{8,8,2} = 0, x_{8,8,3} = 0, x_{8,8,4} = 0, x_{8,8,5} = 0, x_{8,8,6} = 0, x_{8,8,7} = 0,$
 $x_{8,8,8} = 0]]$

```

> for week from 1 to m do
  f:=(i,j)->x[i,j,week]:
  Matrix(n,n,f):
  print(week, subs(Sol[2],Matrix(n,n,f)));
end do:

```

$$1, \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$6, \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$7, \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$8, \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(12)